Work

and

Energy

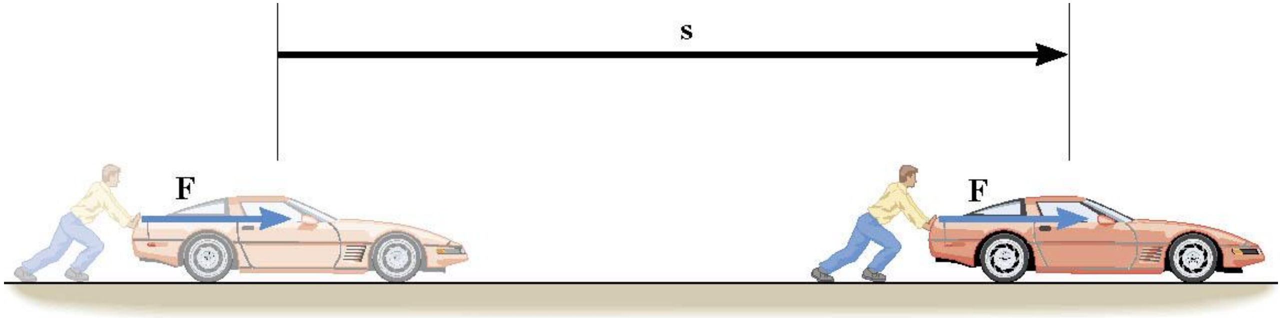
Work changes Energy

Do Work Son!

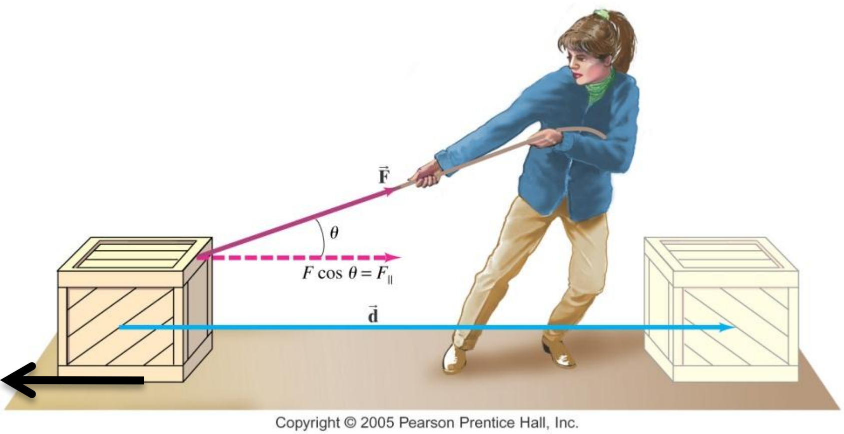
[Do Work Son!]

Work – Energy Relationship

- 2 types of energy
  - kinetic : energy of an object in motion
  - potential: stored energy due to position or stored in a spring
- Work and energy are inseparable – two sides of the same coin
- Energy = ability to do work
- Work
  - process of transferring energy from one object to another
  - process of converting energy from one type to another



- Work done by a constant force
  - product of force component parallel to displacement vector and the displacement

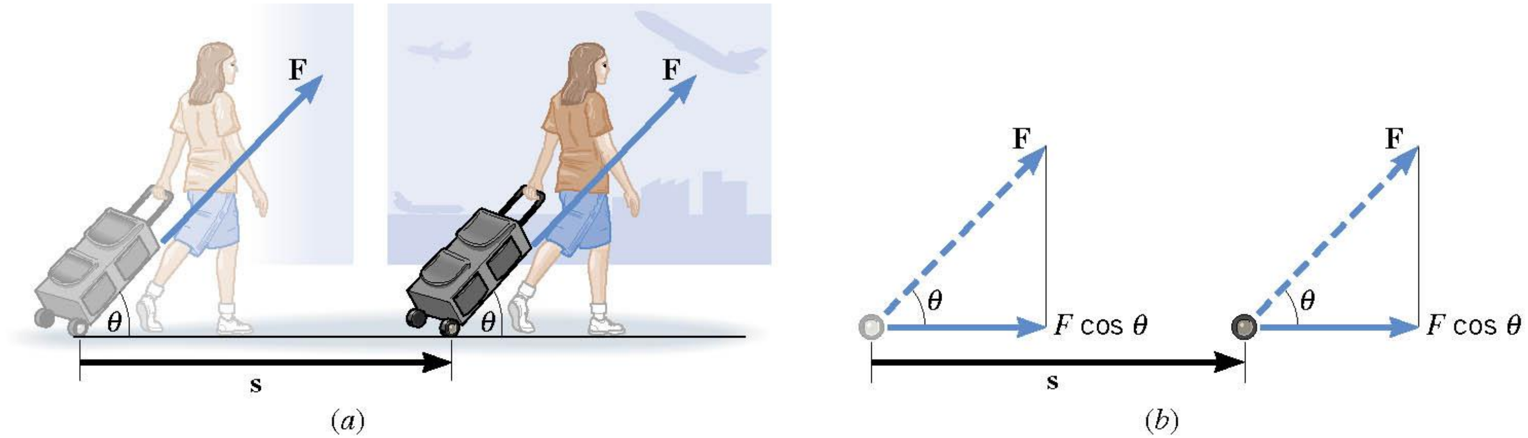


$$Work = F\cos\theta(d) = \vec{F} \bullet \vec{d}$$

SI units = N•m = Joule (J)

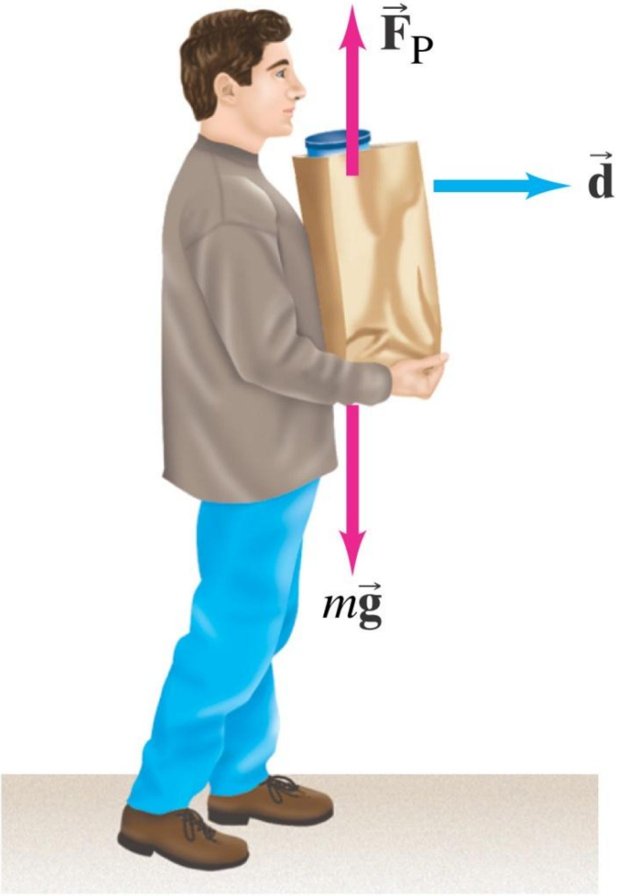
- + work when component of force is in same direction as the displacement
- – work when force is opposite to displacement
- 0 work when force is perpendicular to displacement

Work causes energy change



- Work is done:
  - when an object moves as a result of applied force(s)
  - by the component of the force parallel or anti-parallel to the displacement vector
- Work (scalar quantity) on an object can be:
  - Positive: increases its energy
  - Negative: decreases its energy
  - Zero: no energy change

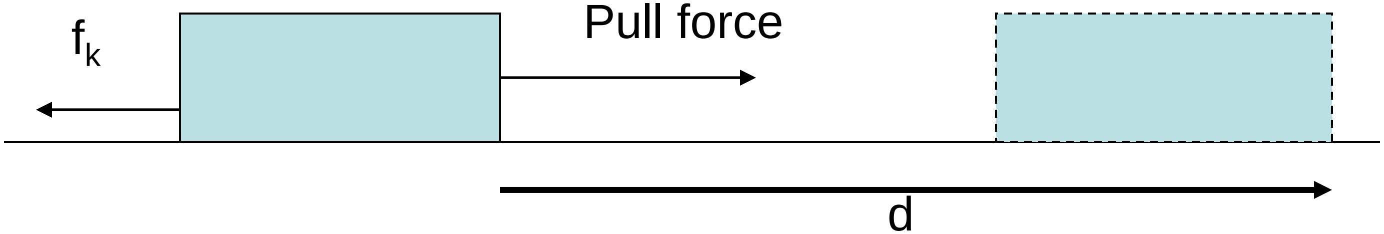
Forces perpendicular to motion do zero work



Net Work

- Sum of work values done on an object
- Can be calculated either by
  - summing up all work values (+, – and 0)

  OR
  - Net force • displacement



1) Net work = (Pull • d) + ($-f_k$ • d) OR

2) Net work = ΣF • d

Work by gravitational force

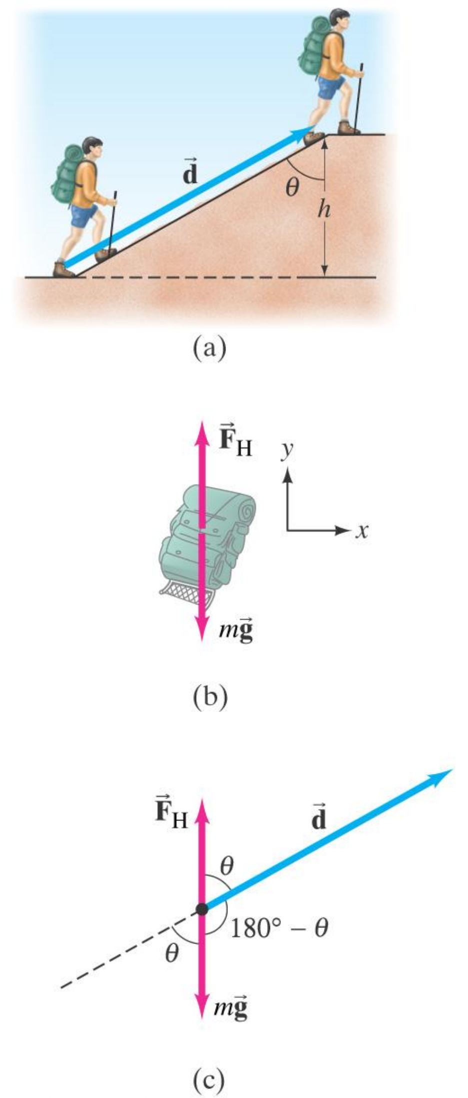
• Work is done by the hiker's force in the vertical direction only

• Use the vertical height that hiker is moving through

• assumes constant speed

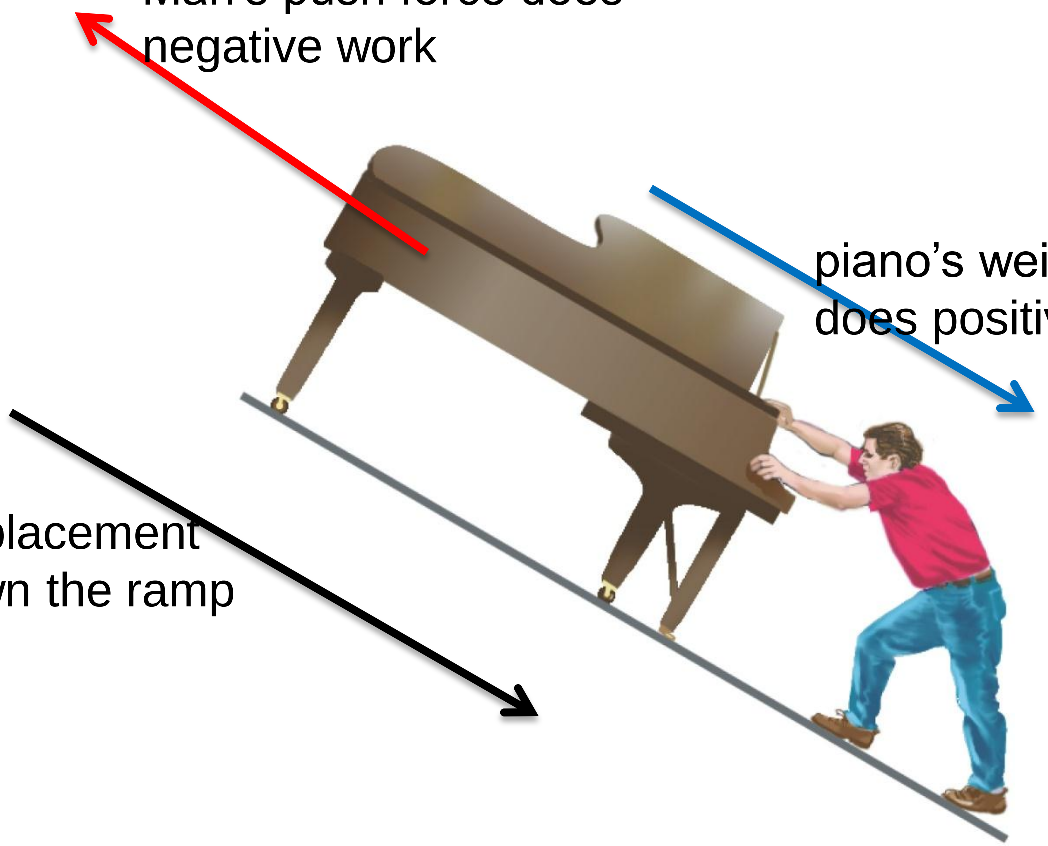
• work by hiker is +

• work by weight force is –



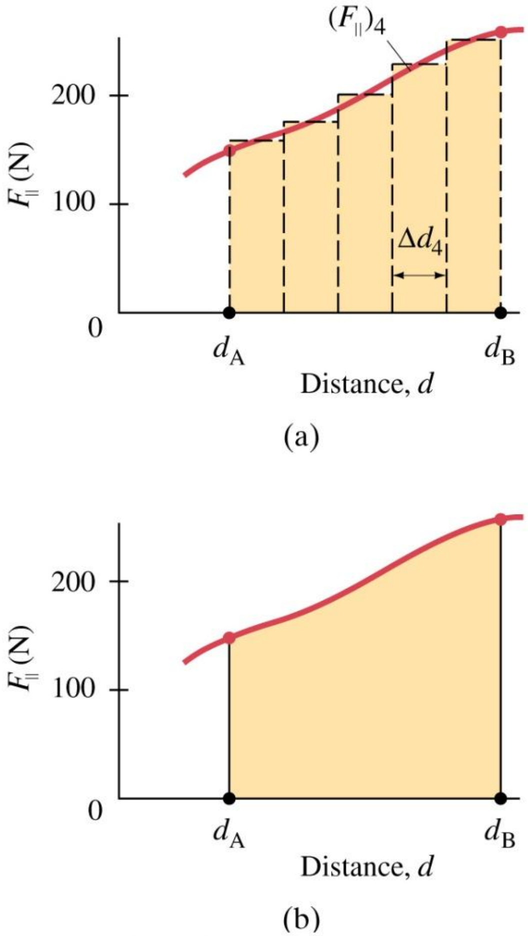
Positive, negative zero work values

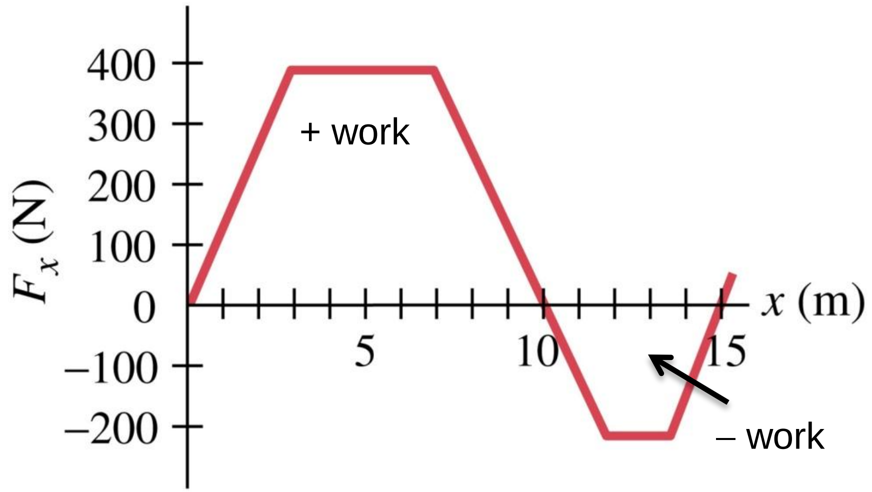
Net work done on piano moving at constant speed = ?
Zero



How can work done by a varying force be determined?

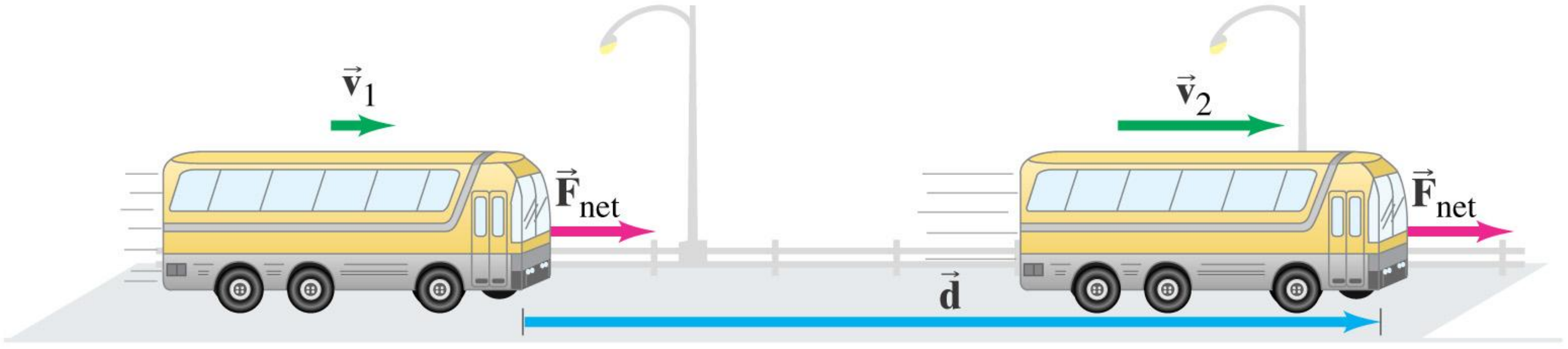
Work = area under Force – displacement graph

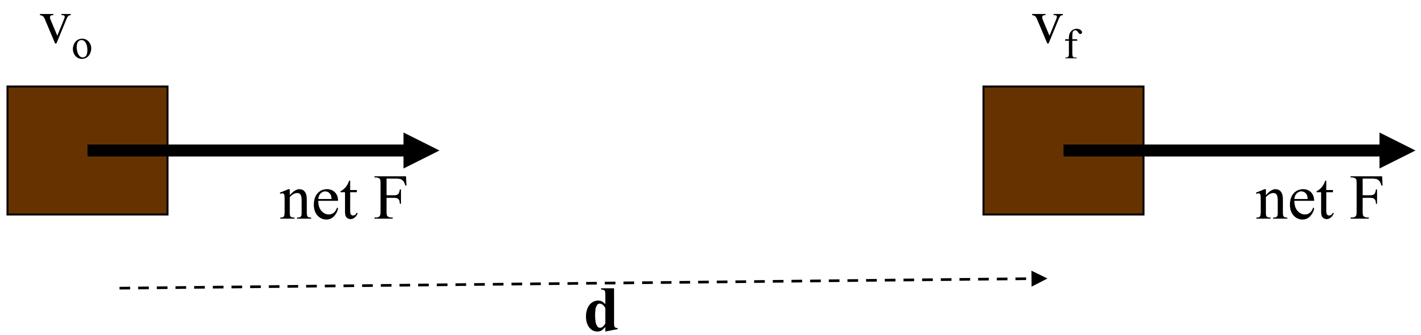




Work – Kinetic Energy Theorem

Net work on an object changes its kinetic energy





$$W_{net} = \Sigma F \bullet d = (ma)d$$

$$W_{netF} = m\left(\frac{v_f - v_o}{t}\right)\left(\frac{1}{2}\{v_f + v_o\}t\right)$$

$$W_{netF} = \frac{1}{2}mv_f^{\ 2} - \frac{1}{2}mv_o^{\ 2}$$

Work

changes

energy

Kinetic Energy

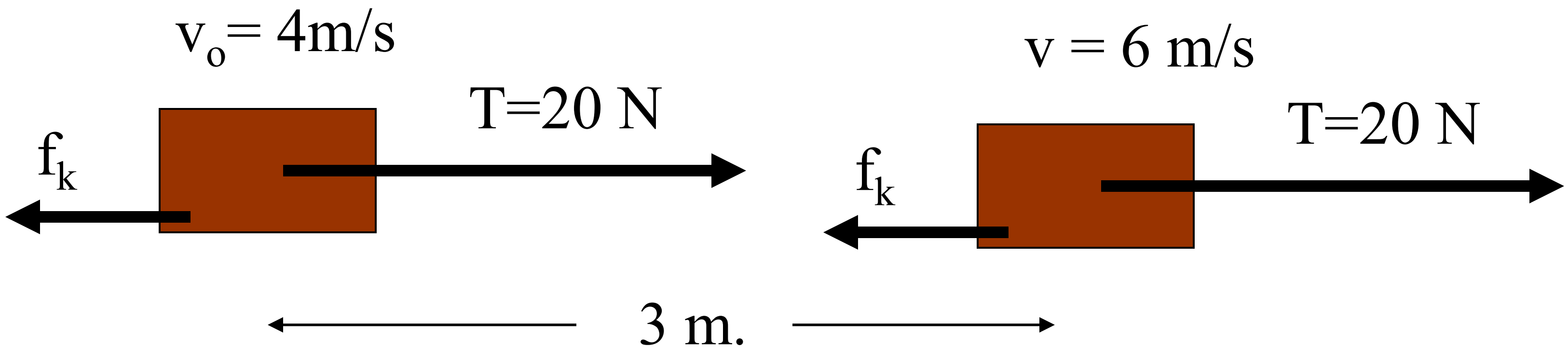
- Kinetic Energy of an object in motion is defined

$$K = \frac{1}{2}mv^2$$

- Total work done on the object to get it from rest to some final velocity
- Kinetic energy is always a positive number – scalar quantity

$$W_{net} = \Delta K = K_f - K_i$$

- Change in Kinetic energy can be positive or negative depending on the change in velocity



What was the change in the KE for our 4 kg mass?

$\Delta KE = 1/2(4)(6)^2 - 1/2(4)(4)^2$

$\Delta KE = 72 - 32 = 40$ J

Find the value of f_k:

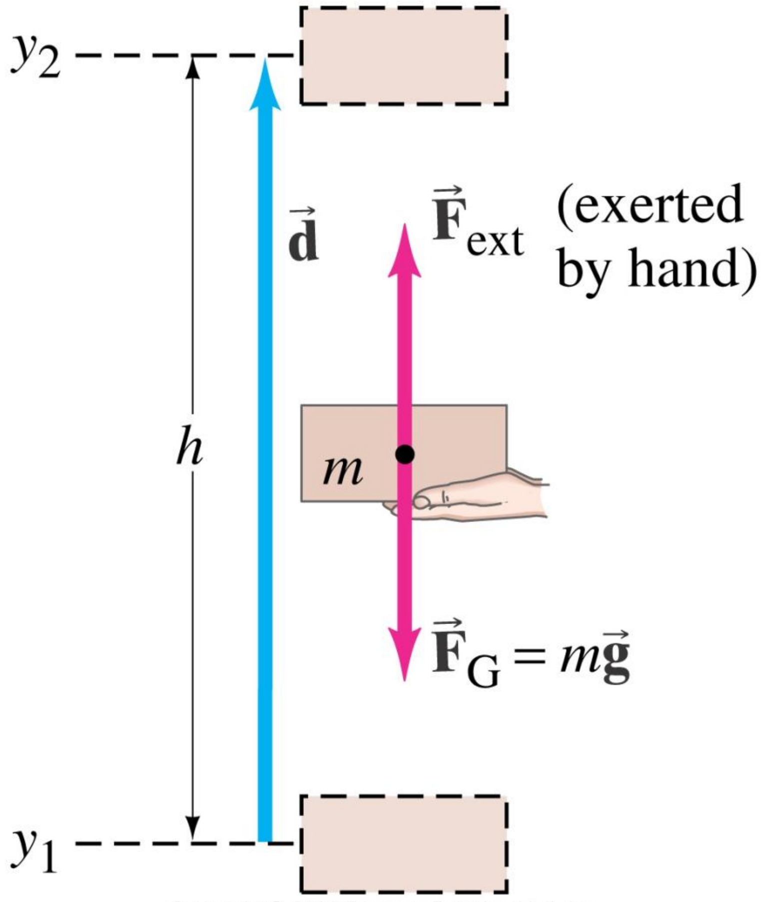
$$W_{total} = \Delta KE$$

$$20(3) - f_k(3) = 40$$

$$f_k = 6.7\ N$$

Gravitational potential energy

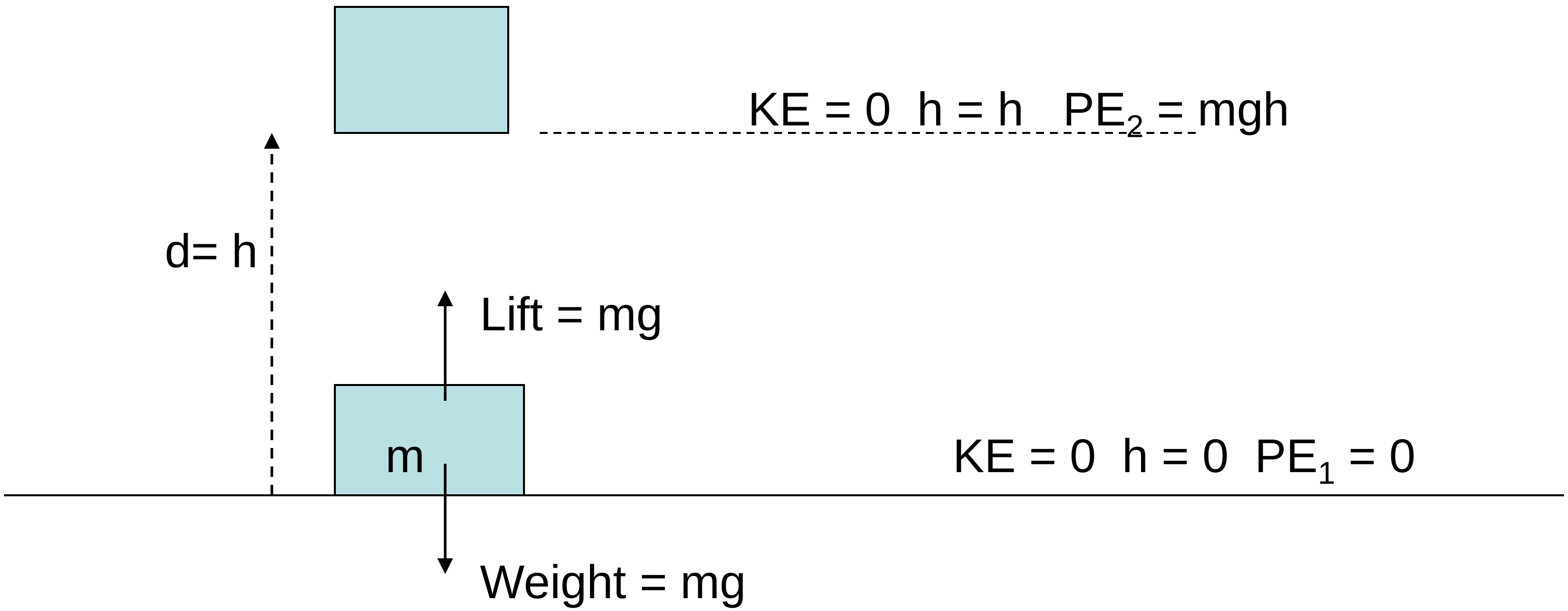
Work by external force = $\Delta PE_{grav} = PE_2 - PE_1$



Gravitational Potential Energy PE

- Work done by an external force on an object, against the gravitational weight force, to change its vertical position, (height = h)
- Requires a system of two objects – one being moved and the Earth which creates gravitational forces on it.
- Defined with respect to a zero energy reference point. h = 0, PE = 0
- Units are Joules
- Stored energy – ability to perform work as a result of its vertical position above/below zero.

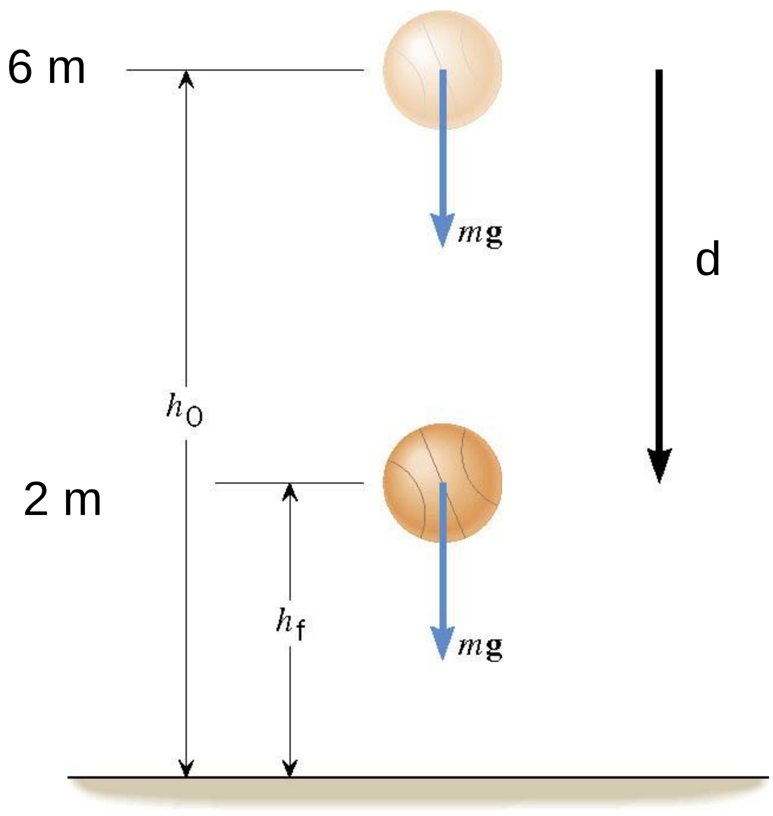
Gravitational Potential Energy



- Lifting from h=0 a vertical distance h is done starting at rest and ending at rest and moves at a constant speed therefore $\Delta KE = 0$
- All work done changes the PE of the object
- Lift force = weight force
- Work = $mg \bullet h = \Delta PE = PE_f - PE_0$
- PE = mgh   (AP equation sheet use U_g for gravitational PE

Work by gravitational forces = – ΔPE

2 kg ball



Initial PE = 2 •9.8 •6 = 117.6 J

Final PE = 2 •9.8 •2 = 39.2 J

Δ PE = - 78.4 J

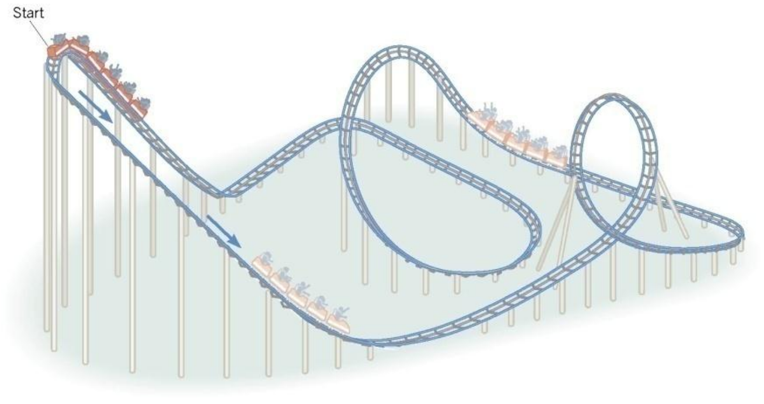
Ball lost 78.4 J of PE

Where did it go?

Gravitational force is a conservative force

- Two criteria define conservative forces

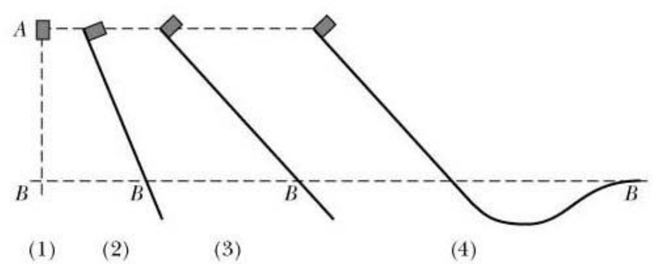
1. Net work done by conservative force around closed loop equals zero
    - Roller coaster has the same amount of energy when it returns to the starting point
    - All PE converted to KE is converted back to PE
    - No energy is converted to heat (“lost” or “left behind”)
    - + in one direction, – in the other direction so the net sum = 0

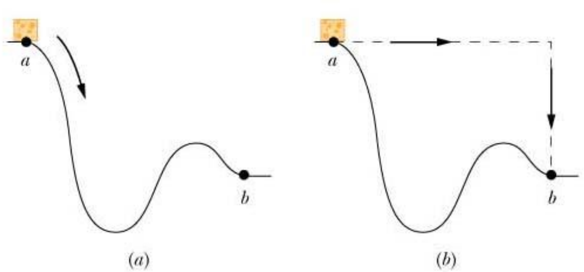


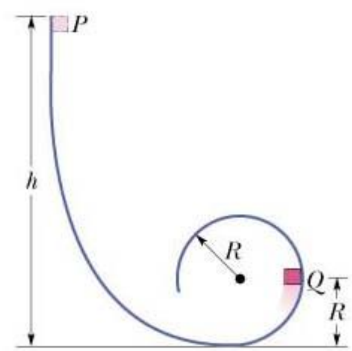
2nd Criteria for Conservative Force

2) Net work is independent of path taken from start to finish;

- Work depends only on the endpoints – where it starts and where it finishes
- Vertical height change between starting and ending points is what determines energy change not path taken

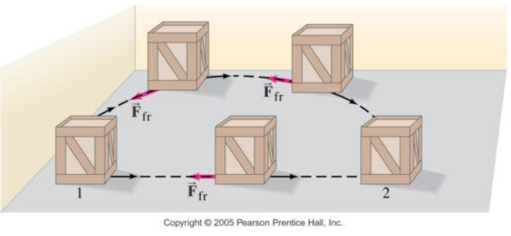


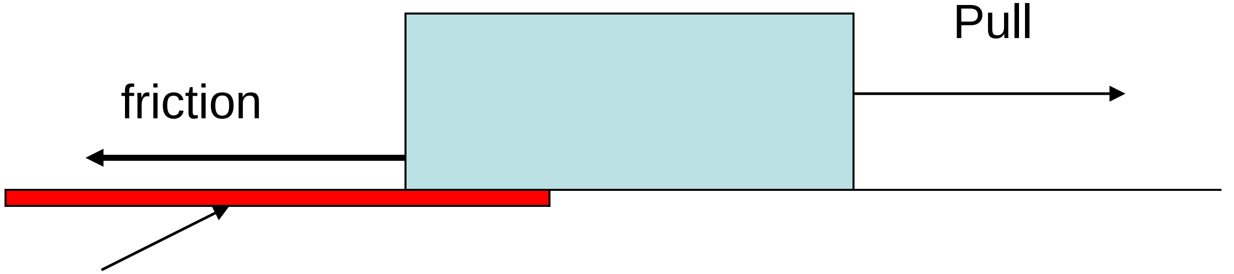




Non-conservative forces

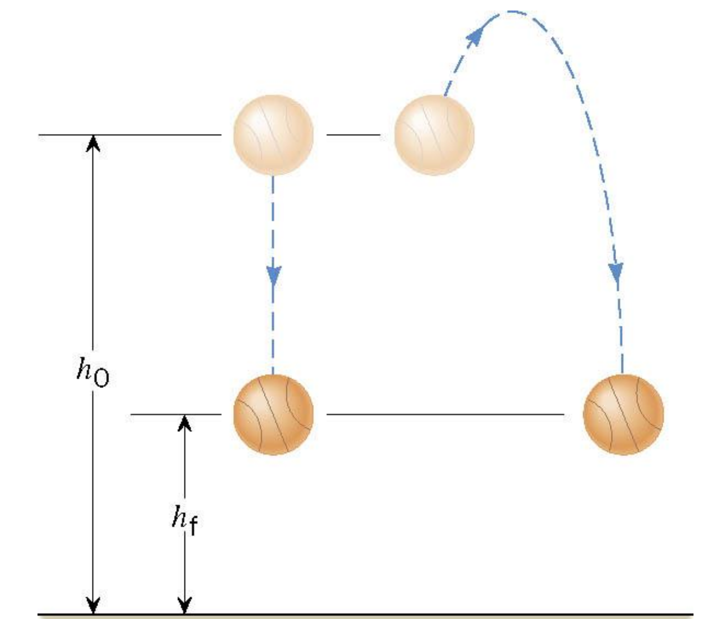
- Friction, air resistance, applied forces from ropes, muscles, motors
- Roller coaster has less kinetic energy when it returns to the start when friction is present
- Negative work done converts energy into heat, not kinetic or potential energy
- If path is reversed the energy lost due to non-conservative forces is not returned to the object



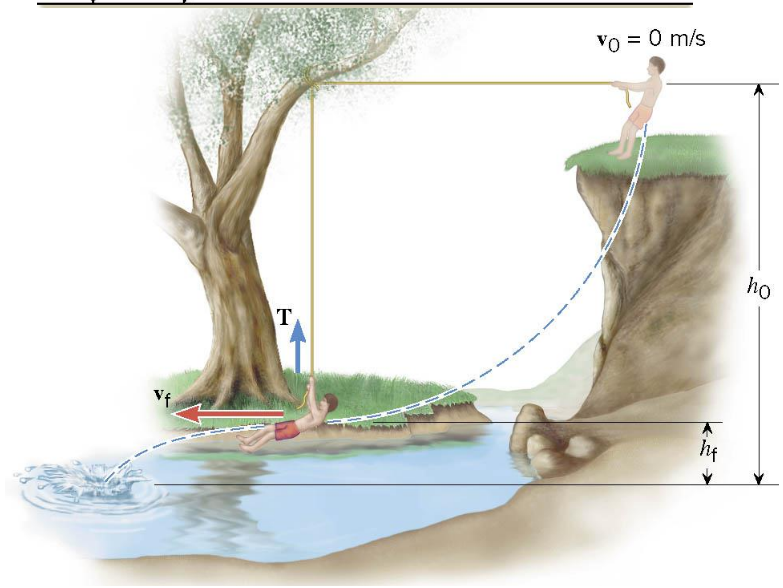


negative work done by friction transfers heat to the floor which cannot be reclaimed by the object

Change in PE is path independent



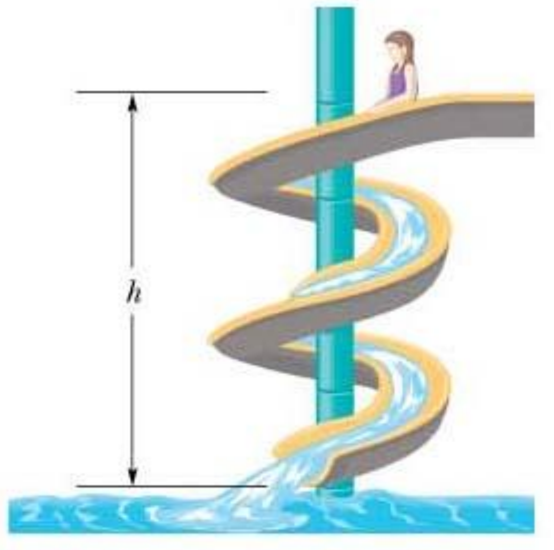
Since gravitational force is a conservative force the ball will lose the same amount of PE whether it falls straight down or moves through the parabola from h_0 to h_f

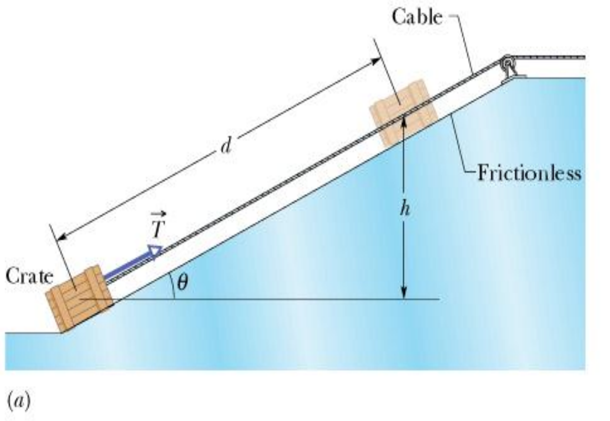


The boy's change in PE is the same when he swings down the semicircular path as when he jumps straight down from the cliff.

Change in PE depends ONLY on the vertical height difference that the mass moves through

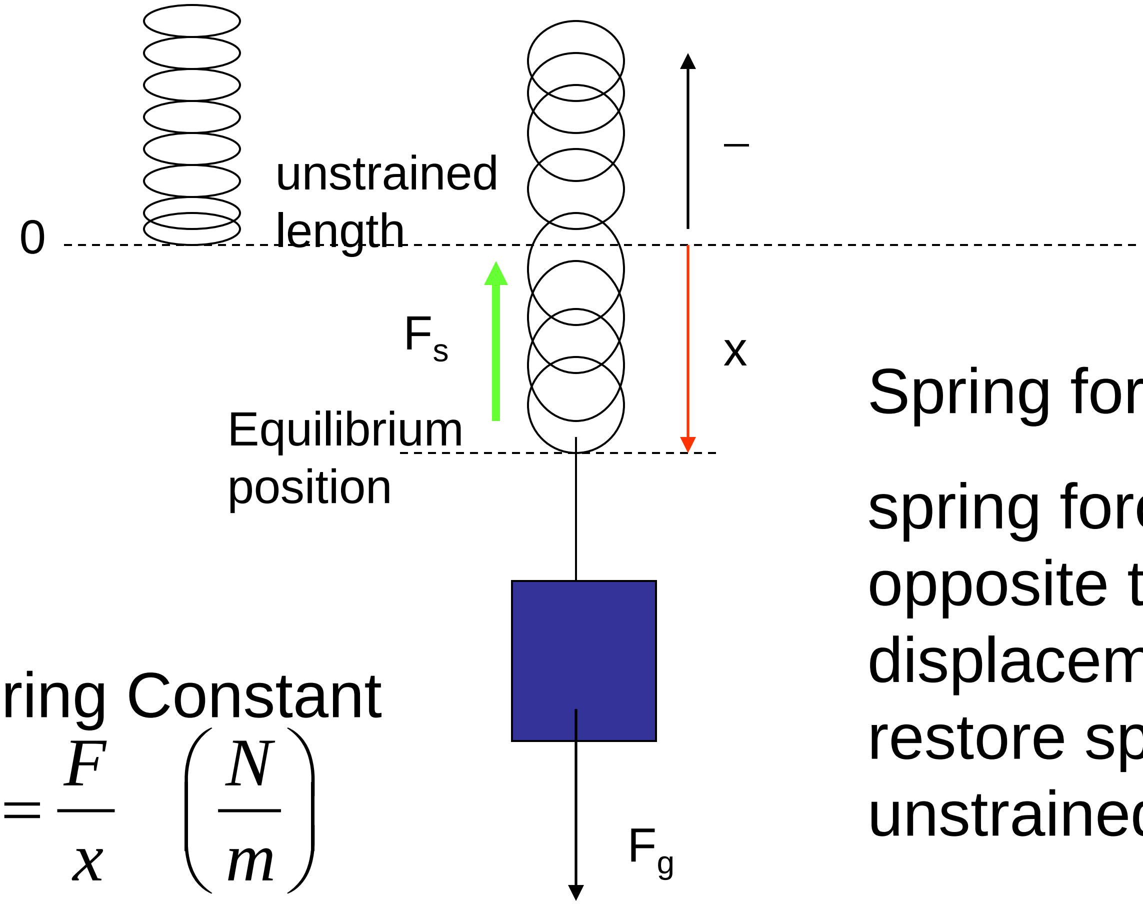
Change in PE is path independent





In the absence of non-conservative forces the work done by gravitational forces (= Δ PE) depends only on the straight line vertical height difference the object moves through

Springs



Spring force F_s=–kx

spring force is always opposite the displacement trying to restore spring to its unstrained length

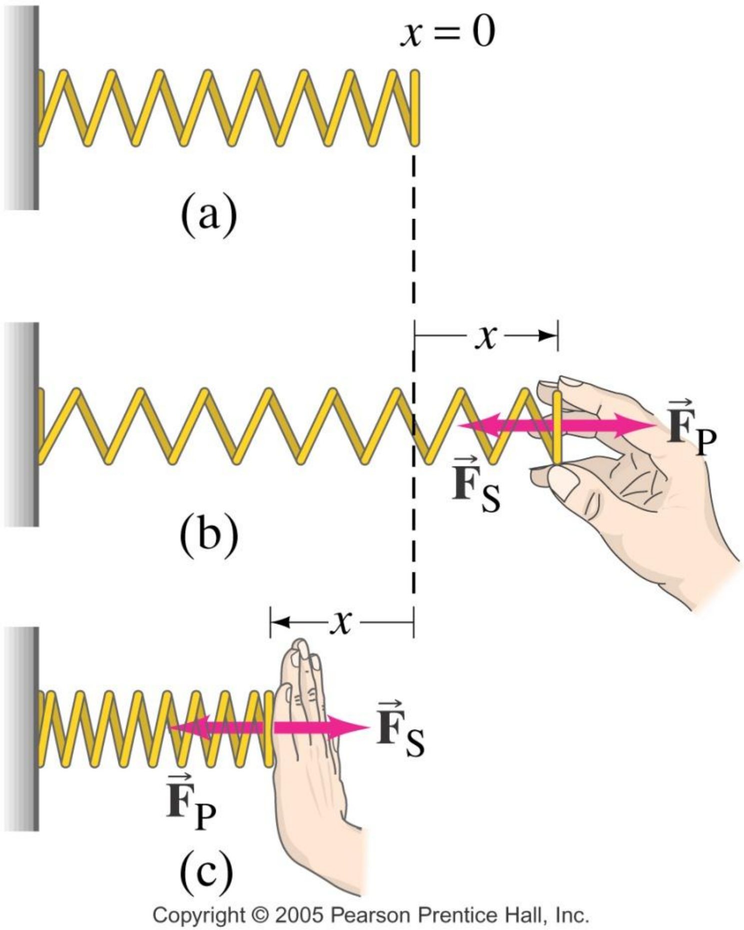
Spring Constant

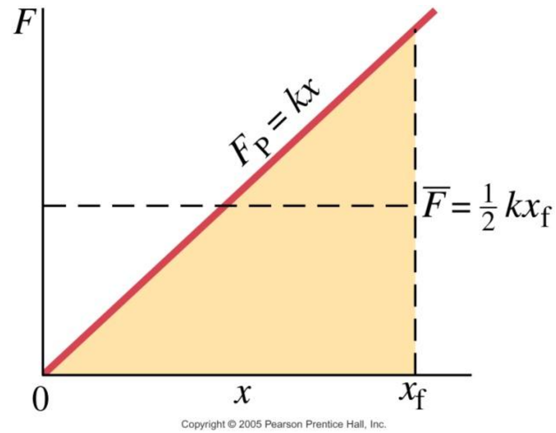
$$k = \frac{F}{x} \quad \left(\frac{N}{m}\right)$$

Spring constant can be determined by hanging a weight on a vertical spring or applying a force to a horizontal spring and measuring the displacement

Elastic Potential Energy stored in spring

Work by external force = $\Delta PE_s = PE_2 - PE_1$



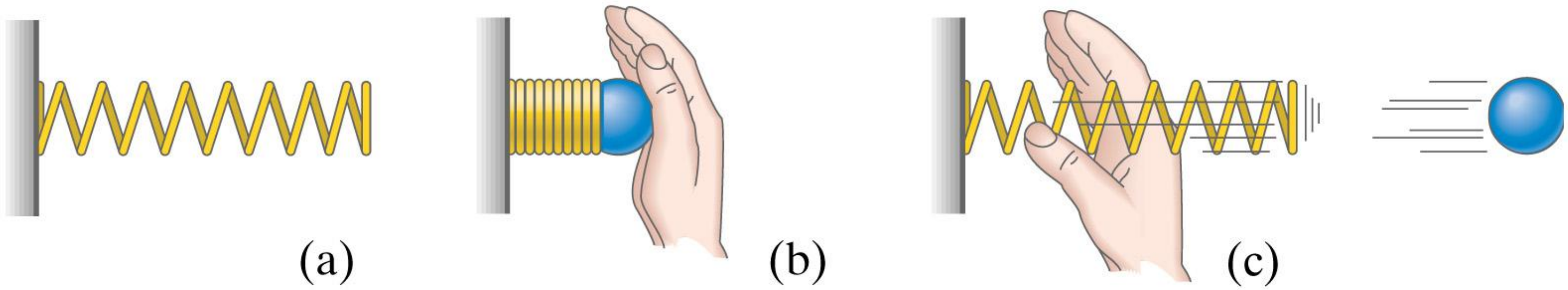


Spring force is variable force but can average force can be used to calculate work

Elastic PE = ½ kx•x = ½ kx^2

AP equation sheet uses U_s

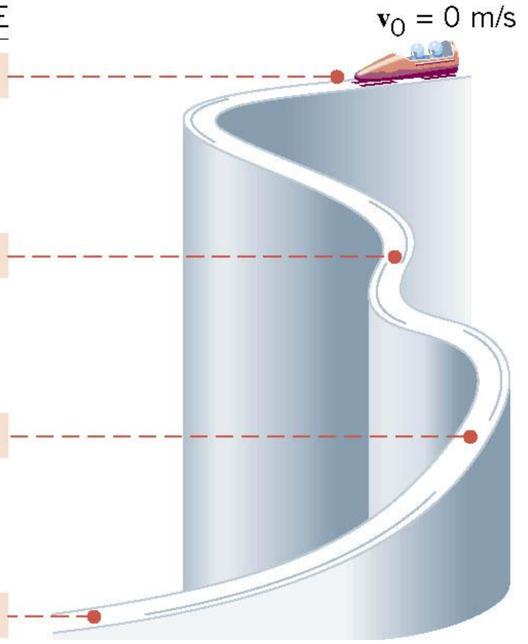
PE stored in spring enables it do work on the ball changing the ball’s KE



Conservation of Mechanical Energy

- Total energy stays constant when no non-conservative forces are present

KE	PE	E = KE + PE
0 J	600 000 J	600 000 J
200 000 J	400 000 J	600 000 J
400 000 J	200 000 J	600 000 J
600 000 J	0 J	600 000 J



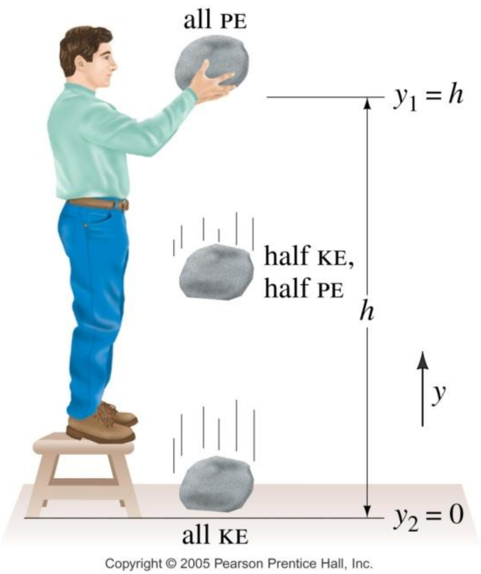
Law of Mechanical Energy Conservation

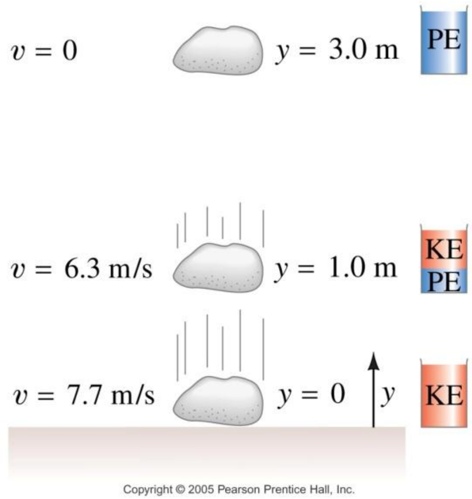
- Energy is neither created nor destroyed it is transferred or converted from one form to another
- Assumes
  - system is closed: no mass added or removed
  - system is isolated: no external forces present
- Only work by non-conservative, external forces can change energy
  - Net work = ΔKE
  - Net work = ΔPE

- Net work= ΔKE+ ΔPE
- since there are no external forces in an isolated system then net work = 0
- $0 = \Delta KE + \Delta PE = (KE_f - KE_0) + (PE_f - PE_0)$
- regrouping initials and finals
- $KE_0 + PE_0 = KE_f + PE_f$
- Total mechanical energy defined as E
- Total E before an “event” = Total E after the “event”
- Total energy change = 0

Total Mechanical Energy stays constant

$$\frac{1}{2}mv_0^{\,2} + mgh_0 = \frac{1}{2}mv_f^{\,2} + mgh_f$$





Only 1 object in problem so mass will cancel

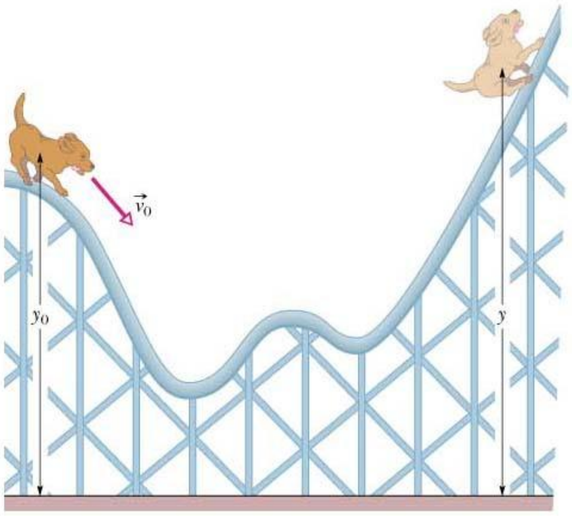
If PE or KE are required then a mass value must be included

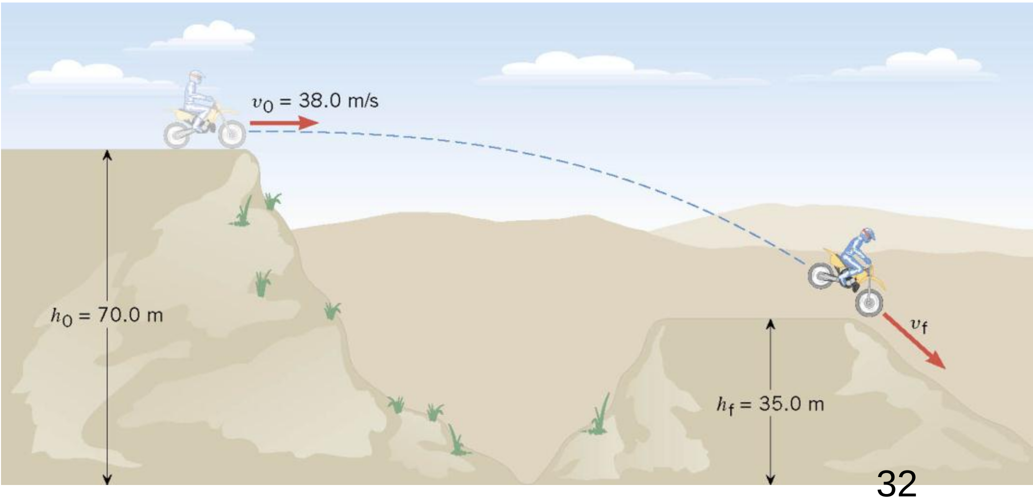
Very useful formula to remember when object has all KE converting to all PE or vice versa:

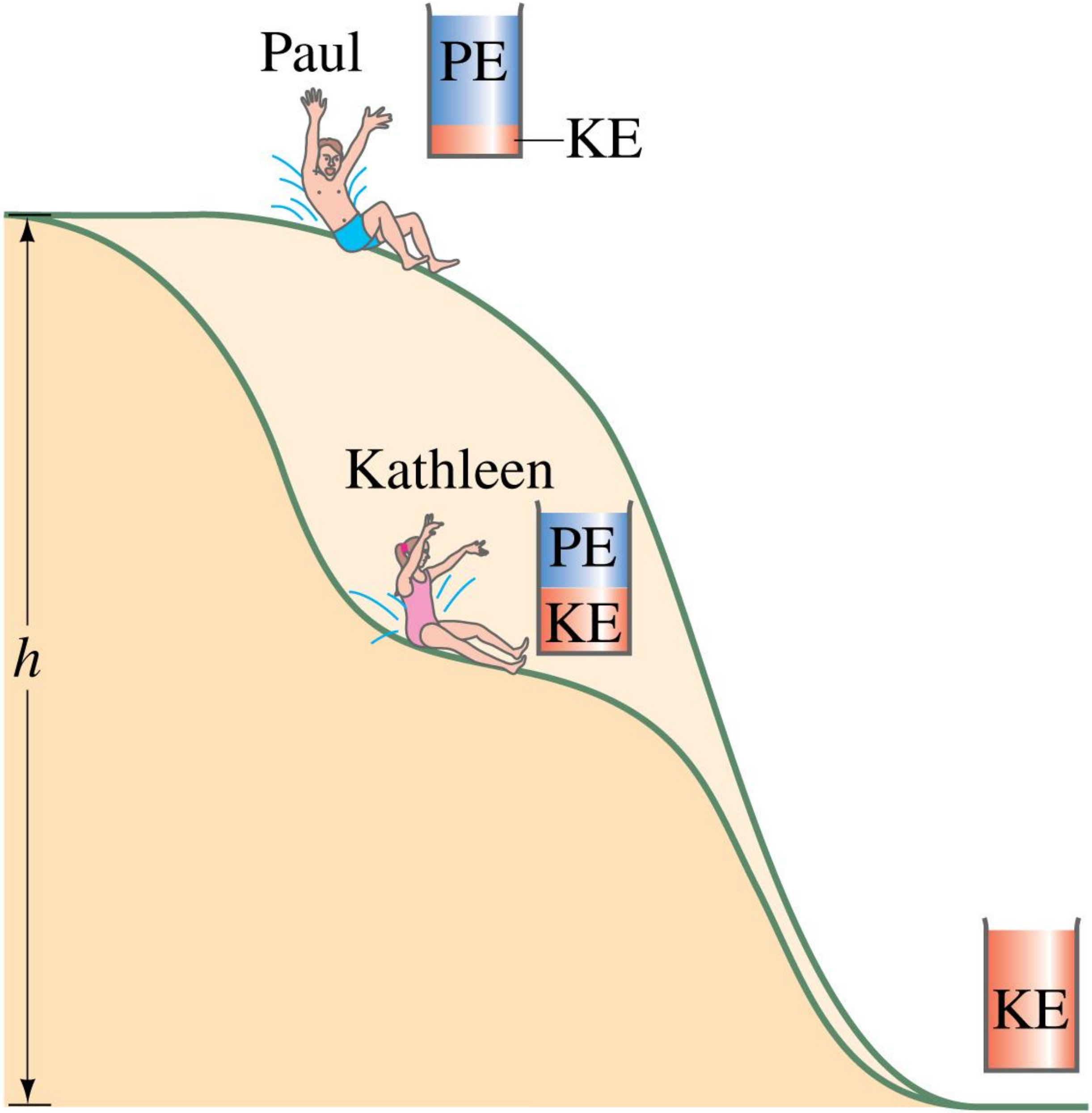
$$v = \sqrt{2gh}$$

Total Energy Stays Constant

- Path independence is major advantage to energy approach
- Simplifies complex path problems
- Only the vertical height change of the object is what determines the amount of energy converted between KE and PE
- No non-conservative forces can be present (friction or air resistance)






Paul
PE
KE
Kathleen
PE
KE
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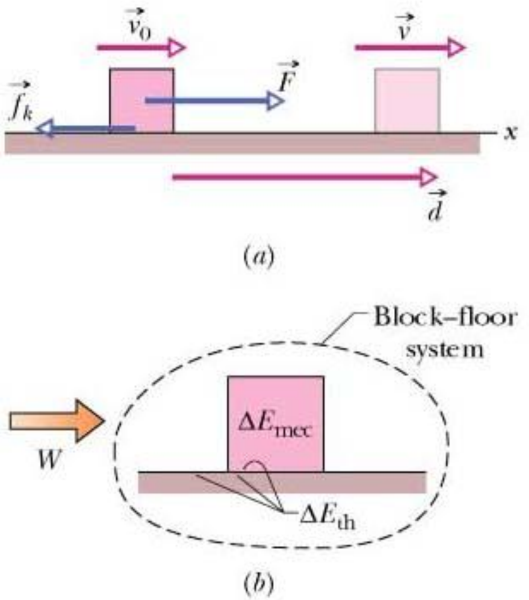
Conservation of Energy Law is not violated when external or non-conservative forces act



Work done by non-conservative forces must be included in the total of energies

Net work = $\Delta K + \Delta U$

$F_{ext} \bullet d - f_k \bullet d = \Delta K + \Delta U$

$Work_{ext} = \Delta K + \Delta U + (f_k \bullet d)$

$\Delta E_{thermal}$

Must account for work or energy change associated with all forces

- conservative or non-conservative
- external or internal to system

Power

- Rate of change of work with time
- Time rate of energy transfer

$$Power = \frac{Work}{time} = \frac{Force \bullet dis\tan ce}{time}$$

$$Power = Force \bullet average\ velocity$$

- Units

$$1\ \text{Watt (W)} = \frac{1\ \text{Joule}}{1\ \text{second}}$$

Energy transfer from power

$$Power = \frac{energy\ transfer}{time} \qquad Energy\ transfer = power \bullet time$$

Efficiency of a motor or heat engine

$$\text{efficiency} \quad e = \frac{\text{power output}}{\text{power input}} = \frac{\text{work output}}{\text{energy input}}$$

all motors or heat engines are less than 100% efficient since some of the input energy is converted to heat or overcomes friction and cannot be used to do work