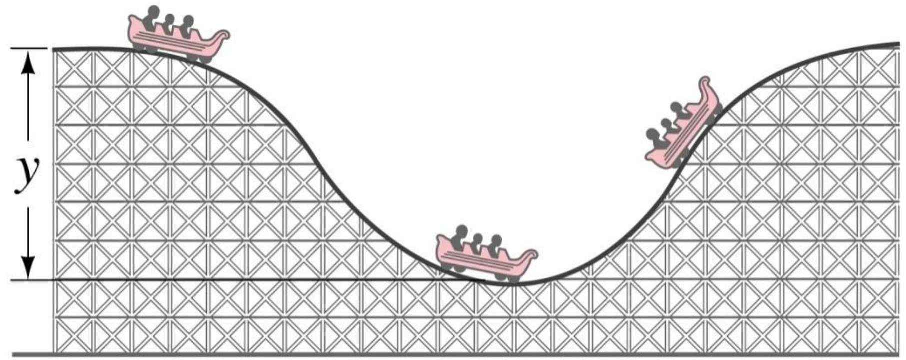
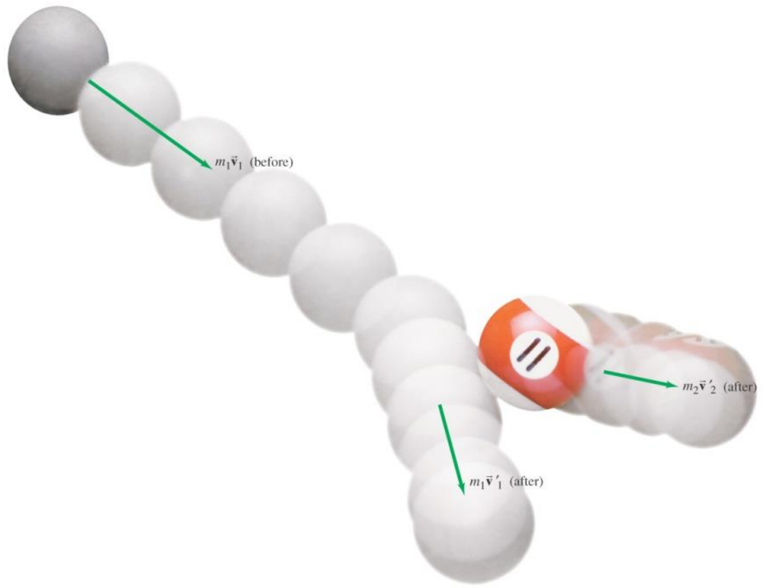
Chapter 7: Linear Momentum

Chapter 6 – energy transformations involved only one object



Chapter 7 – momentum is transferred when 2 or more objects interact in a collision



Linear Momentum

- Product of an object's mass and velocity

$$\vec{p} = m\vec{v}$$

- vector quantity with same direction as velocity vector
- Units:

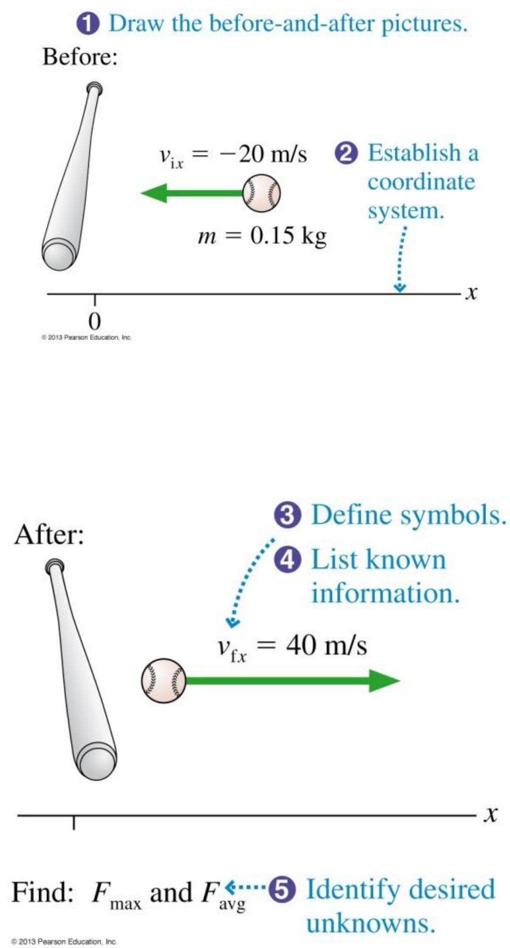
$$kg \bullet \frac{m}{s} = \left(kg \bullet \frac{m}{s^2} \right) \bullet s = N \bullet s$$

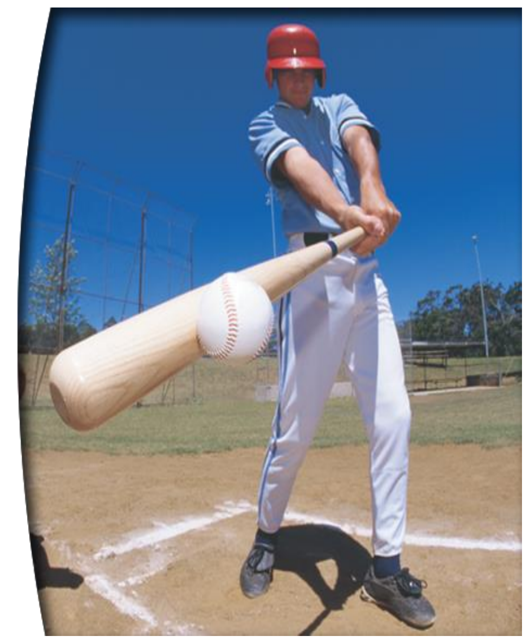
Momentum is a vector quantity

- Newton actually started with the quantity of momentum and how forces changed it

$$\Sigma F = \frac{\Delta p}{\Delta t} = m\frac{\Delta v}{\Delta t} = ma$$

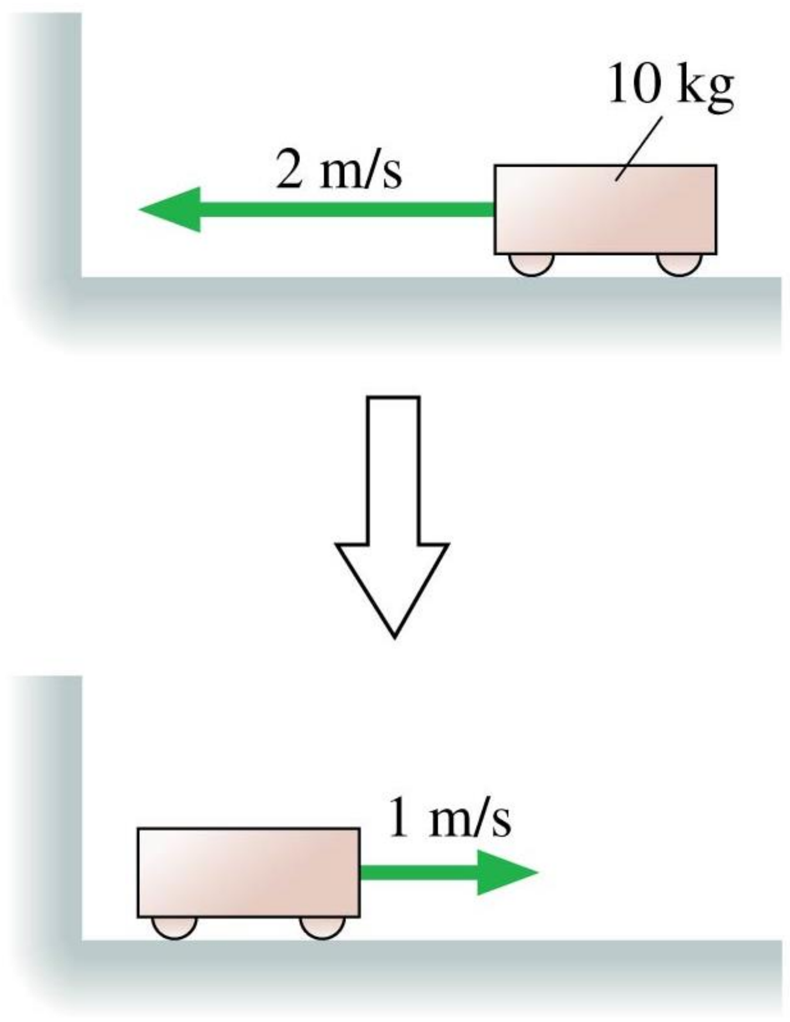
- Bat applies an average force over a short time interval to accelerate the ball
- Must stop the ball, change its direction and send it in the opposite direction
- change in momentum of the ball results
- Δp is also a vector quantity





The cart's change of momentum Δp_x is

A. –20 kg m/s.
B. –10 kg m/s.
C. 0 kg m/s.
D. 10 kg m/s.
E. 30 kg m/s.



The cart's change of momentum Δp_x is

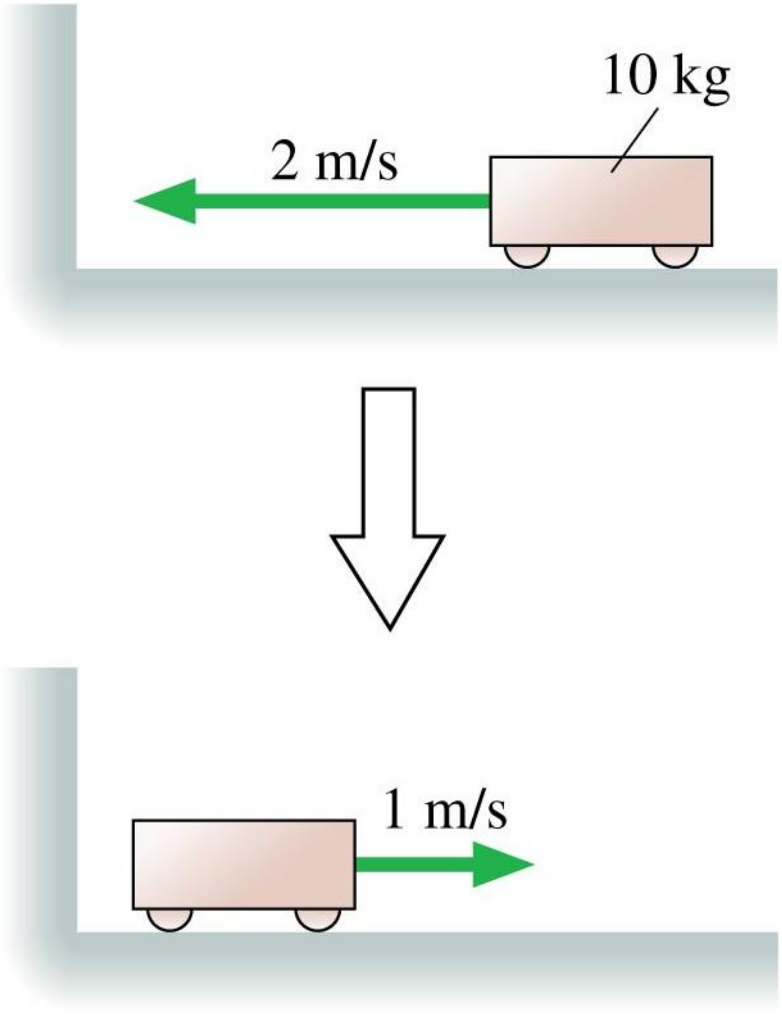
A. –20 kg m/s.

B. –10 kg m/s.

C. 0 kg m/s.

D. 10 kg m/s.

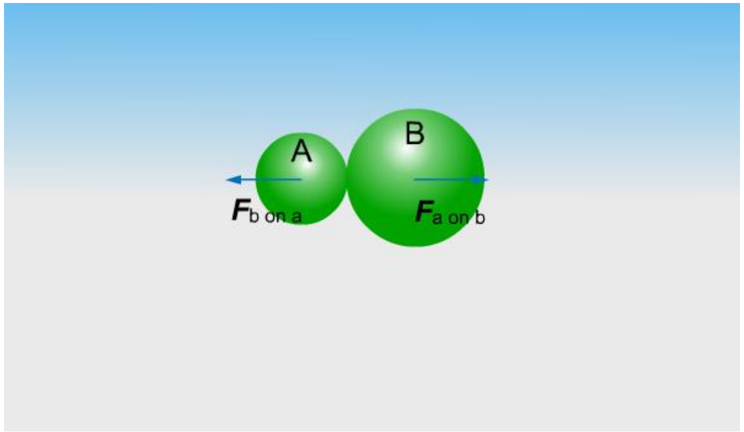
✔ **E. 30 kg m/s.**



Δp_x = 10 kg m/s – (–20 kg m/s) = 30 kg m/s

Negative initial momentum because motion is to the left and $v_x < 0$.

Conservation of Linear Momentum



During collision each ball exerts an equal force on the other in the opposite direction according to Newton's 3rd Law of Motion

Force of A on B changes B's momentum in time interval Δt

Force of B on A changes A's momentum in time interval Δt

$$\vec{F}_A = -\vec{F}_B$$

$$\frac{\Delta \vec{p}_B}{\Delta t} = \frac{-\Delta \vec{p}_A}{\Delta t}$$

Total Linear Momentum stays constant

Time interval Δt is the same for both

$$\Delta\vec{p}_B = -\Delta\vec{p}_A$$

$$p_{Bf} - p_{Bi} = -\left(p_{Af} - p_{Ai}\right) = p_{Ai} - p_{Af}$$

$$p_{Bf} + p_{Af} = p_{Ai} + p_{Bi}$$

$$m_B v_{Bf} + m_A v_{Af} = m_B v_{Bi} + m_A v_{Ai}$$

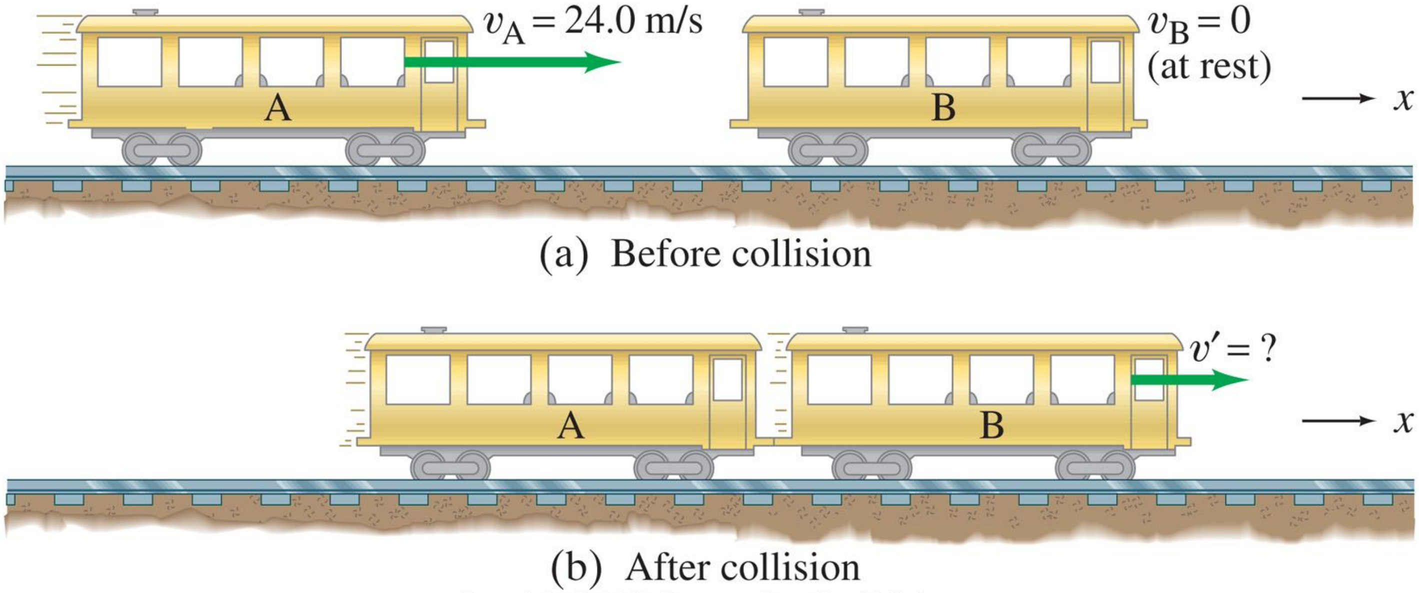
Total momentum after the collision equals the total momentum before the collision.

Assumes that system is closed and isolated

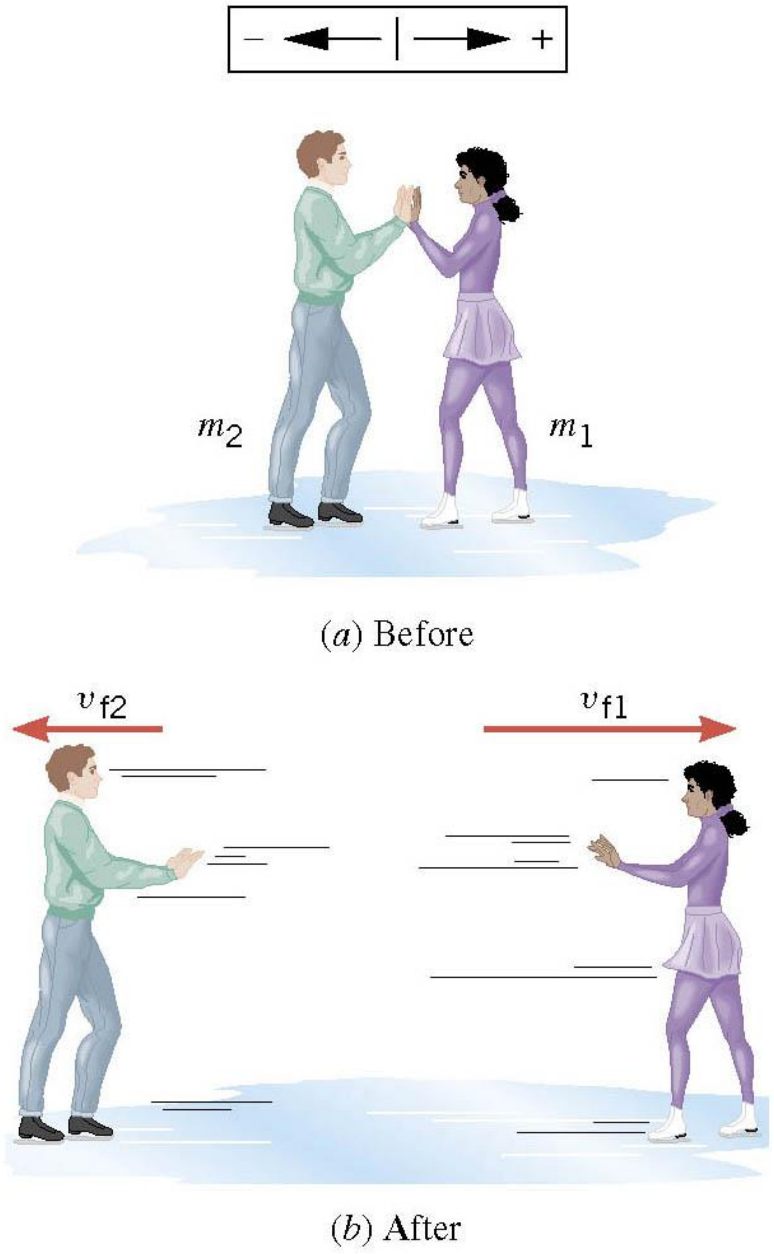
Isolated = no external forces

Closed = no mass or particles being added or removed from system

Conservation of Momentum Examples



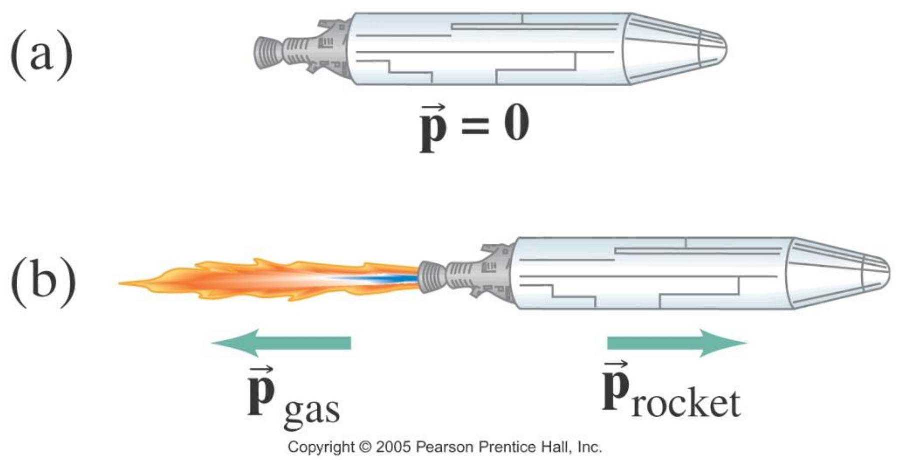
When two particles stick together after the collision there is only one mass (total of $m_1 + m_2$) and only one final velocity v_f

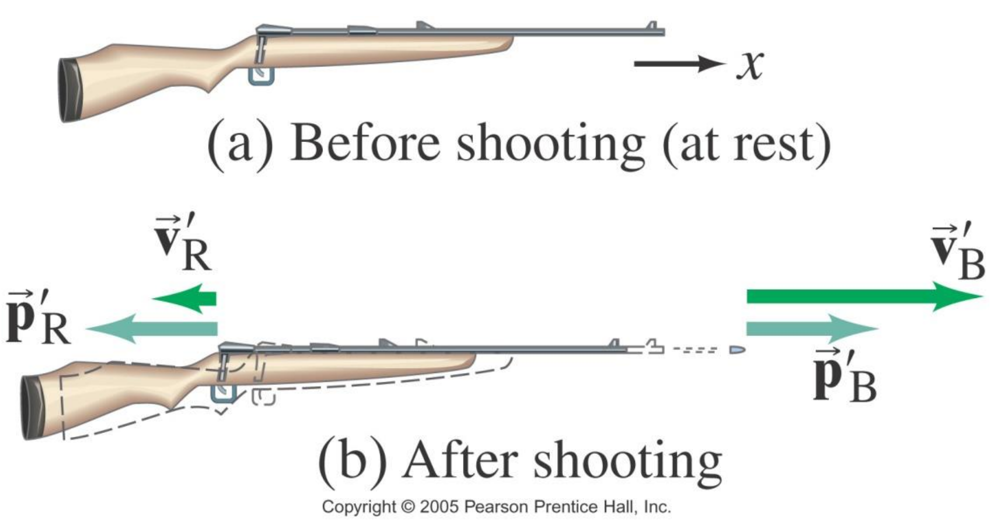


Momentum is a vector

Total momentum before "collision" = 0

Must have + and – signs on the final velocities for total momentum after the "collision" = push to be equal to 0





Total momentum before and after = 0

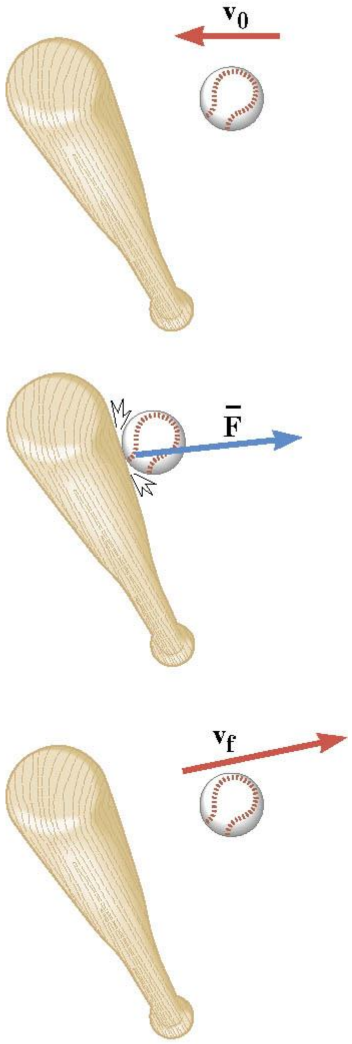
Total KE after the event ?

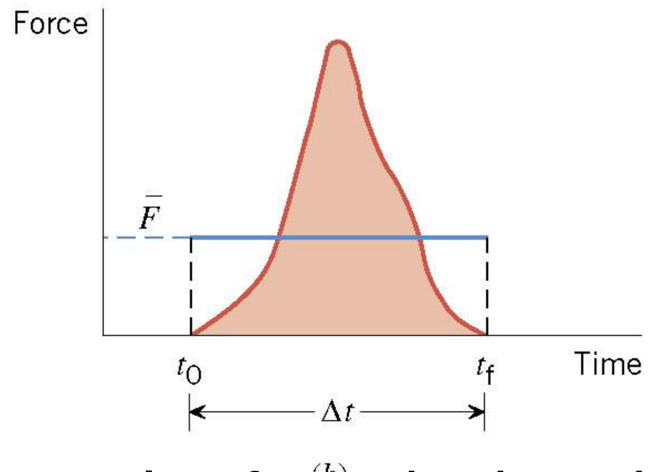
Greater than 0

Source of energy?

Must be able to calculate KE_i, KE_f, ΔKE

Impulse = $F_{avg} \bullet \Delta time$





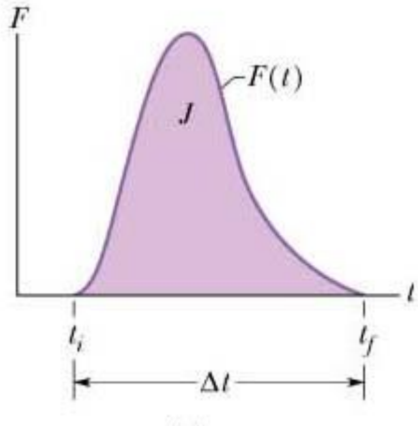
Impulse from the bat changes the momentum of the ball

Actual force builds to a very high maximum value and then drops off

We will most often study forces that have a constant average value

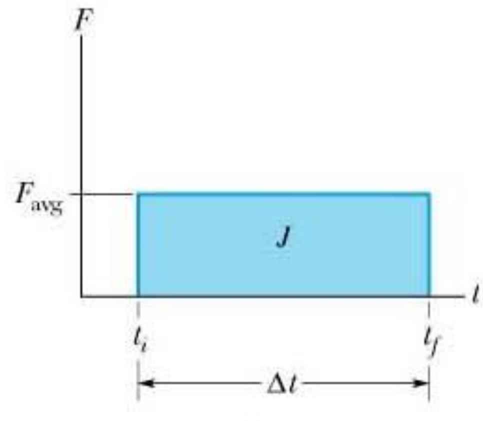
Impulse = area under force – time graph

variable force



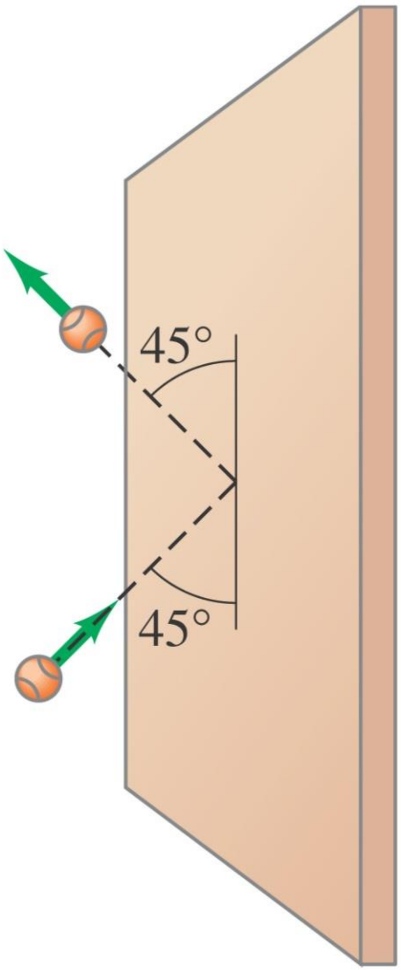
(a)

constant force



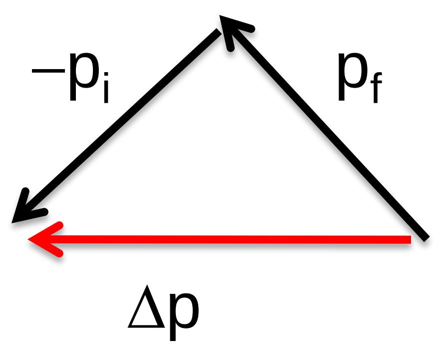
(b)

Impulse = vector quantity in same direction as force vector



- A ball hits a wall and bounces off at the same speed.
- What is the direction of the impulse acting on the ball?

$$\text{Impulse} = \Delta p = p_f - p_i$$



Impulse – momentum theorem

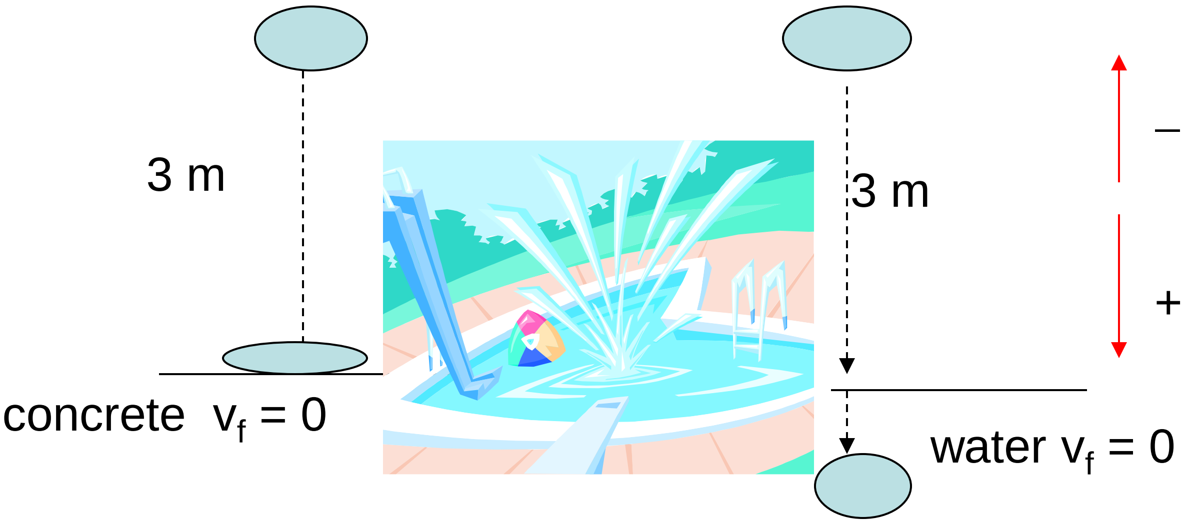
- Impulse = change in momentum

$$\vec{F} \bullet \Delta t = m\Delta\vec{v} = m\vec{v}_f - m\vec{v}_0 = \vec{p}_f - \vec{p}_0 = \Delta\vec{p}$$

- Equivalent or identical?
- equivalence means that if you calculate the impulse you know what the change in momentum is equal to
- equivalence means that if you know the mass and velocity change of the object then you know the value of the impulse that changed its velocity

Airbag-watermelon drop-balloon toss

2 identical 1 kg watermelons are dropped from rest from a 3 meter diving board



conservation of energy to solve for velocity just before collision

$$PE = KE$$

$$1 \bullet 9.8 \bullet 3 = \frac{1}{2} \bullet 1 \bullet v^2$$

$$v = 7.7\ m/s$$

Δp for both cases is the same = 0 – 7.7 kgm/s = - 7.7 kgm/s

Impulses for both water and concrete are both – 7.7 kgm/s

Collision time for concrete Δt = .001 s   Force = - 7700 N (up)

Collision time for water Δt = 1 s   Force = - 7.7 N

TIME IS THE DETERMINING FACTOR, NOT THE FORCE

Application of Impulse – Momentum Theorem

- Airbag principle
- water balloon toss

$$\Sigma \vec{F} \bullet \Delta t = \Delta \vec{p}$$

$$1000\,N \bullet 0.01 s = 10\,N \bullet s$$

$$10\,N \bullet 1 s = 10\,N \bullet s$$

- 2 extremes in changing momentum from large value to zero
- Very high force acting over a very short time
- Lower force acting over a longer time
- Time is what determines the force NOT force which determines the time

A mosquito and a truck have a head-on collision. Splat! Which has a larger change of momentum?

A. The mosquito.

B. The truck.

C. They have the same change of momentum.

D. Can’t say without knowing their initial velocities.

A mosquito and a truck have a head-on collision. Splat! Which has a larger change of momentum?

A. The mosquito.

B. The truck.

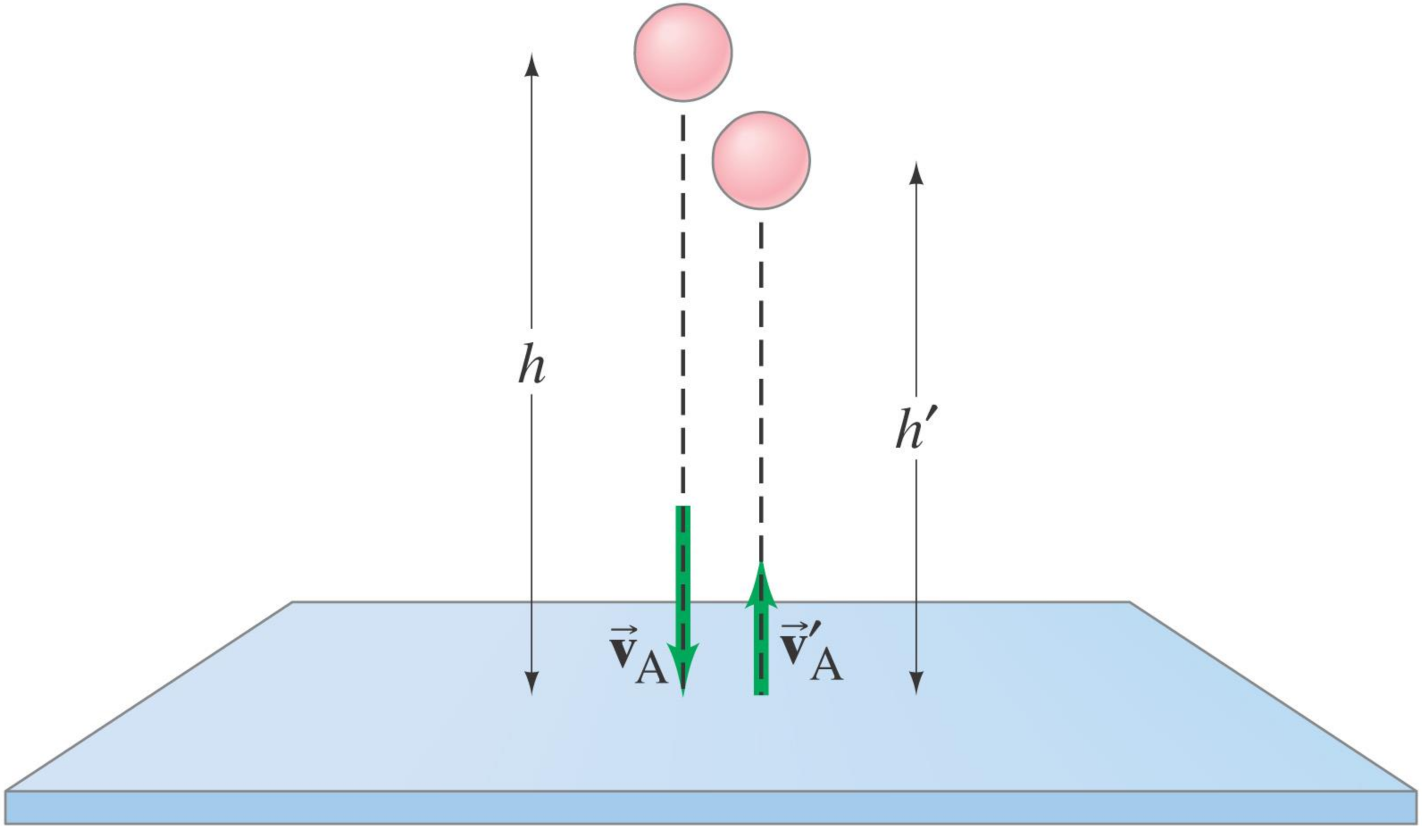
✔ **C. They have the same change of momentum.**

D. Can't say without knowing their initial velocities.

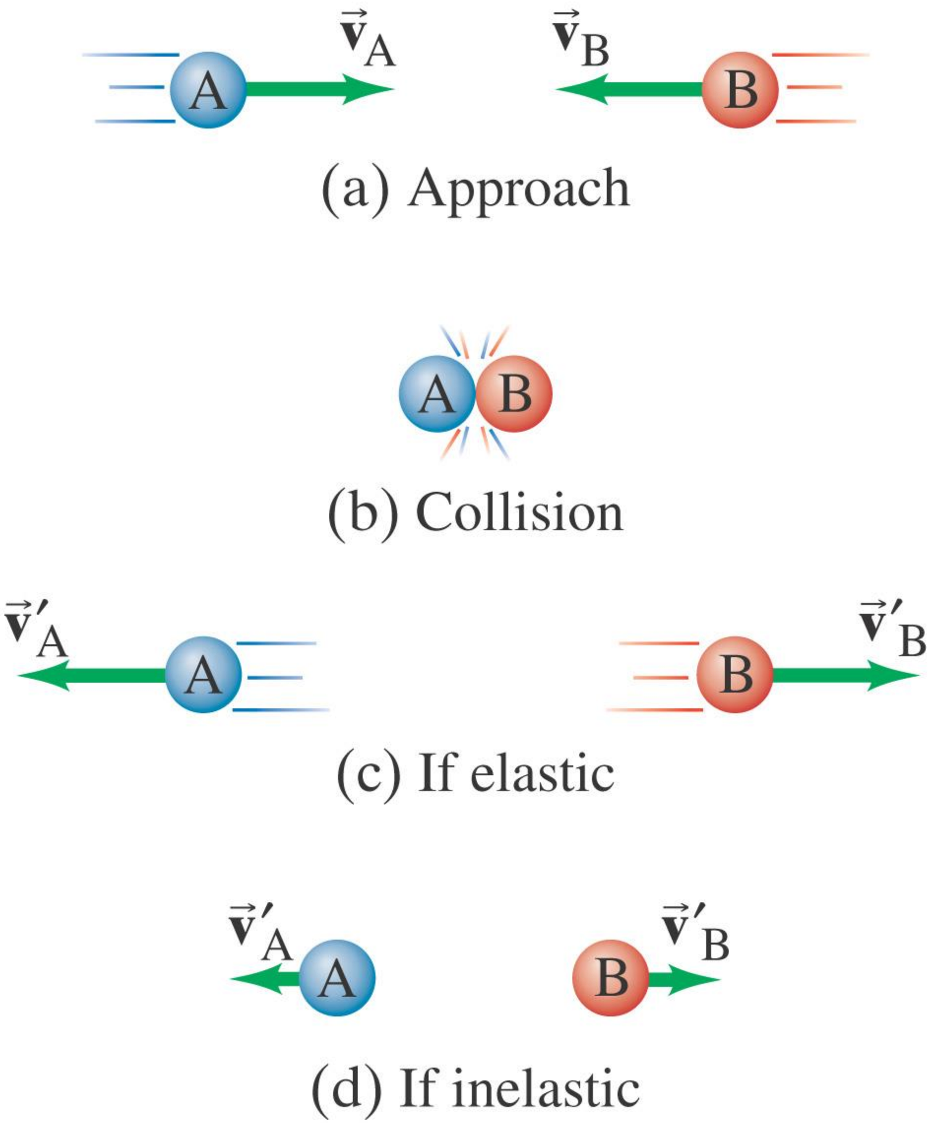
Momentum is conserved, so $\Delta p_{\text{mosquito}} + \Delta p_{\text{truck}} = 0$.

Equal magnitude (but opposite sign) changes in momentum.

Force plate – ball drop example



7-4 Conservation of Energy and Momentum in Collisions



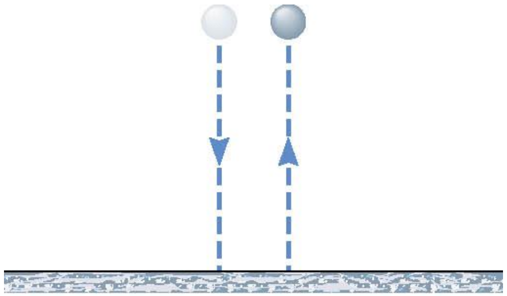
Momentum is **conserved** in all **collisions**.

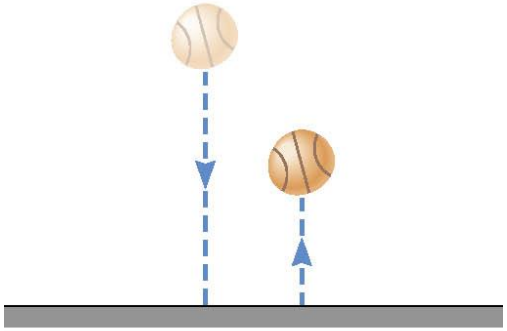
Collisions in which **kinetic energy** is conserved as well are called **elastic** collisions, and those in which it is not are called **inelastic**.

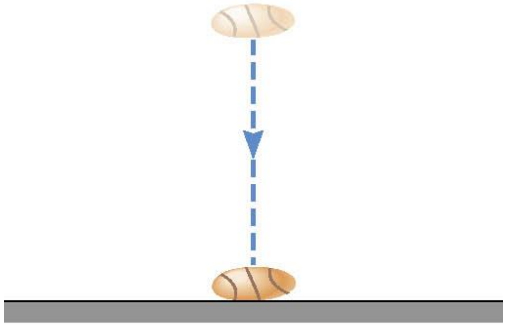
Elastic, Inelastic Collisions

- Momentum is always conserved for all types of collisions, in a closed, isolated system
- However energy has the ability to do work in the form of deformation of one of the objects in the collision or to generate heat
- Collisions are classified by amount of kinetic energy that is “used up” doing work during collision

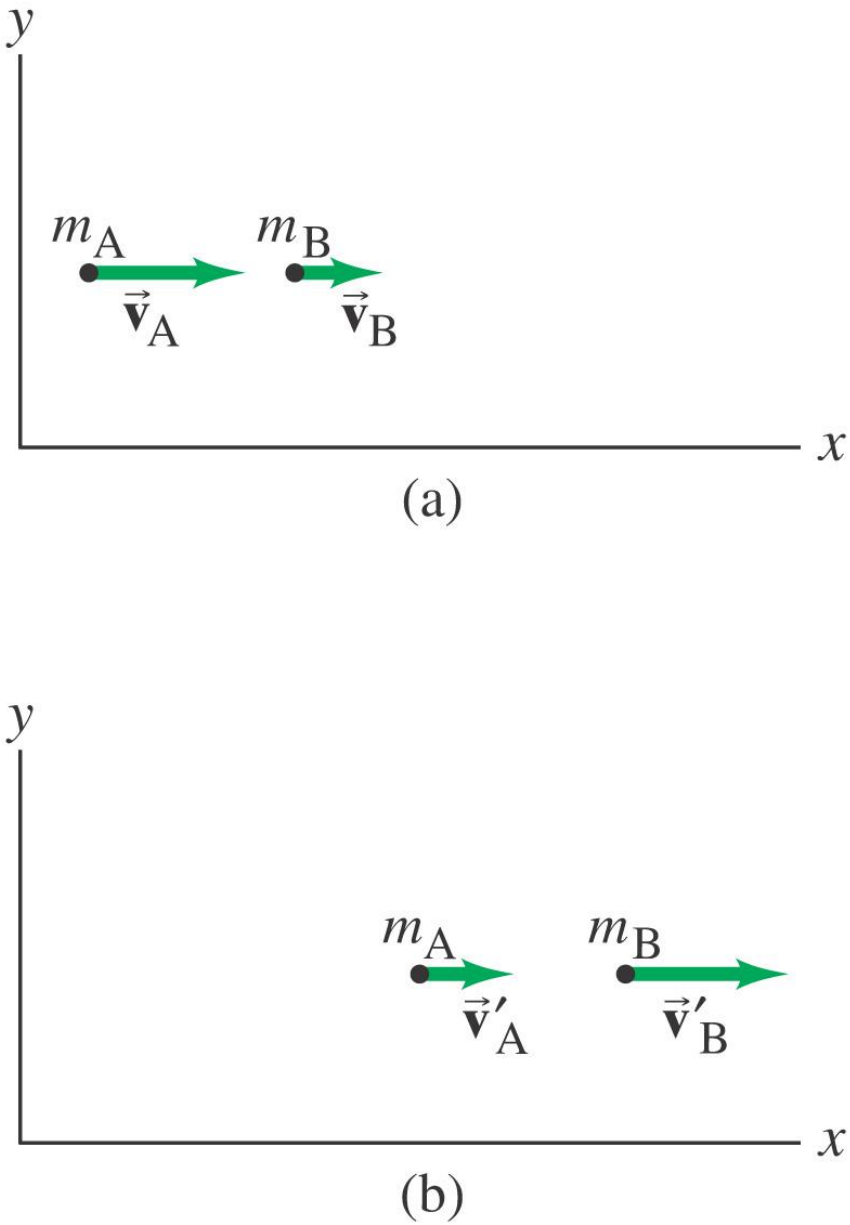
- Completely Elastic
  - total KE before = total KE after
- Inelastic
  - total KE before > total KE after
  - some KE used up to compress and expand the ball during collision
- Completely Inelastic
  - objects stick together after collision
  - may or may not be moving


(a) Elastic collision


(b) Inelastic collision


(c) Completely inelastic collision

7-5 Elastic Collisions in One Dimension

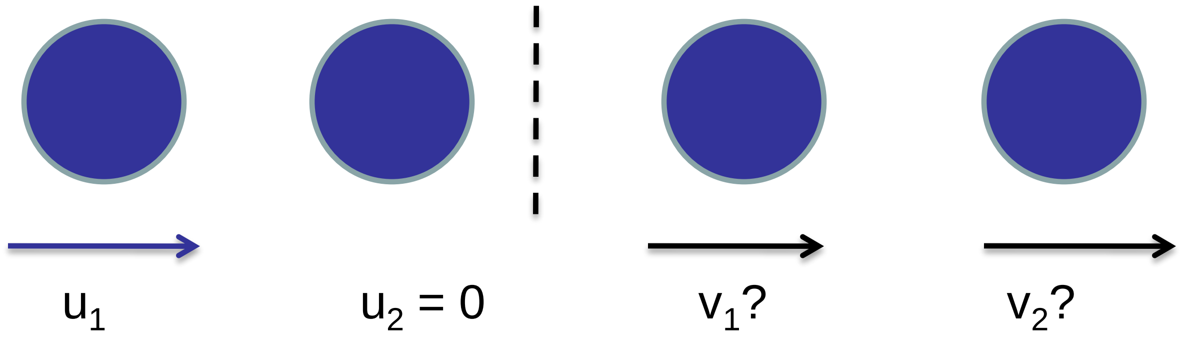


Here we have two objects colliding **elastically**. We know the masses and the initial speeds.

Since both **momentum** and **kinetic energy** are conserved, we can write **two** equations. This allows us to solve for the **two** unknown final speeds.

Head on elastic collision

Two equal mass balls – one moving at initial velocity u_1 towards stationary ball 2. What are the 2 final velocities?



Momentum is conserved

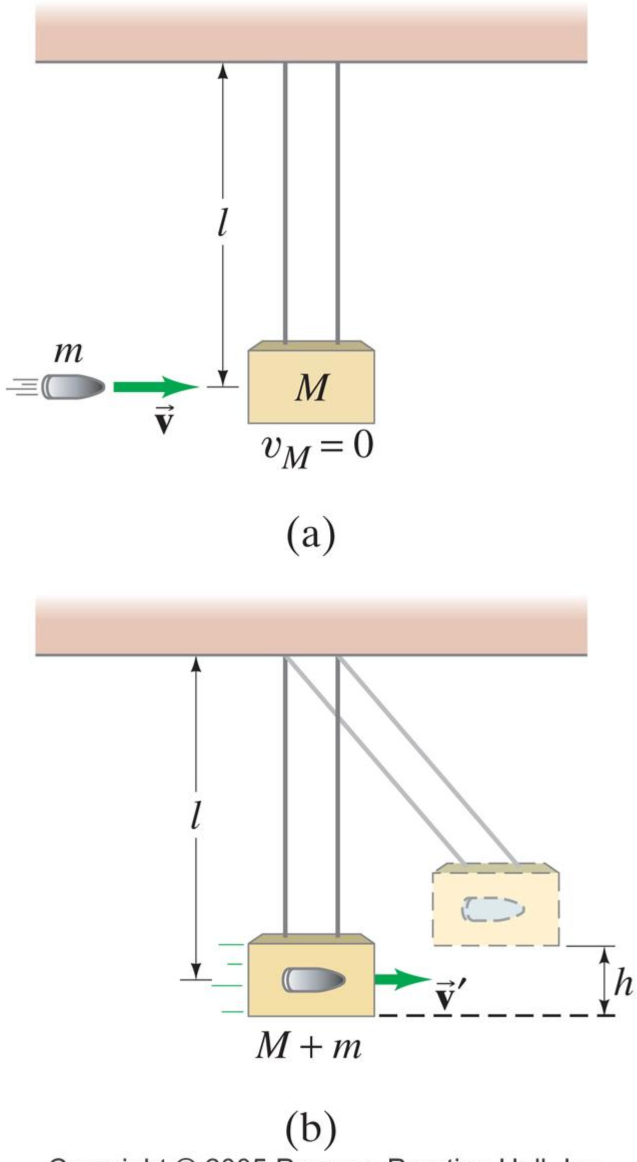
$mu_1 = mv_1 + mv_2$

KE is conserved

$1/2mu_1^2=1/2mv_1^2 + 1/2mv_2^2$

Result: ball 1 stops, ball 2 moves off at initial speed of 1

Ballistic Pendulum



They may be a bullet shot into a block which rises or a ball which swings down and strikes a block which then moves to the right.

Use energy conservation to solve for speed – height change relationship

Use momentum conservation to solve for velocities before/after collision

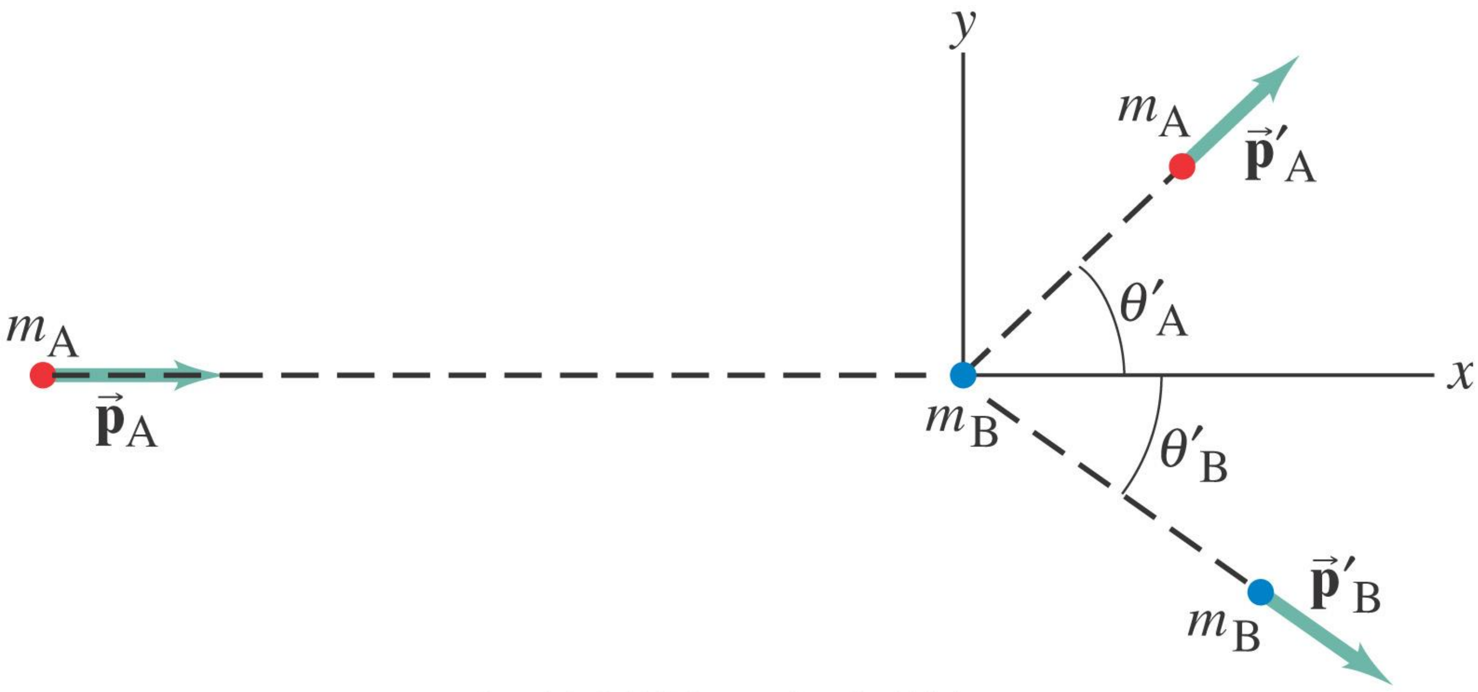
Ballistic Pendulum

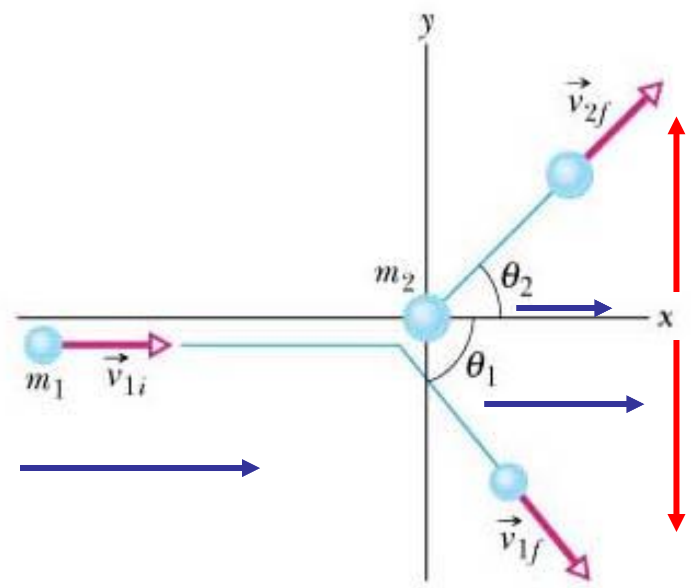
- Completely inelastic collision between bullet and block
- KE of bullet > KE of bullet+block
  - some of bullet's KE used to deform block
- cannot equate final PE of pendulum with initial KE of bullet
- cannot equate initial and final KE's
- momentum is conserved for the collision

2 – dimensional collisions

total initial momentum in x = total final momentum in x

total initial momentum in y = total final momentum in y



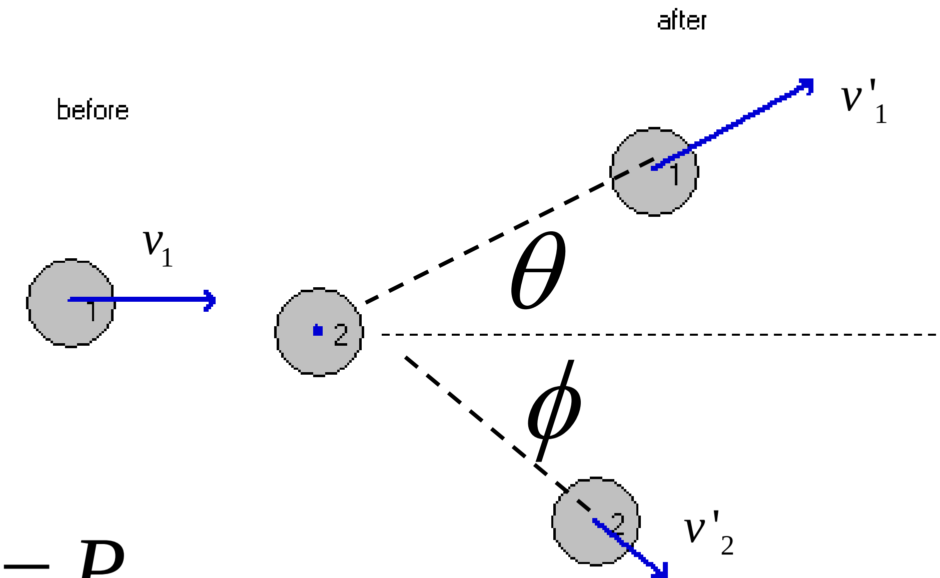


y components of final momenta must be equal and opposite to sum up to zero

total initial momentum of moving object will equal sum of the two final horizontal components

A two dimensional collision

$$\vec{P}_i = \vec{P}_f$$



$$P_{xi} = P_{xf}$$

$$m_1 v_1 = m_1 v'_1 \cos\theta + m_2 v'_2 \cos\phi$$

$$P_{yi} = P_{yf}$$

$$0 = m_1 v'_1 \sin\theta - m_2 v'_2 \sin\phi$$