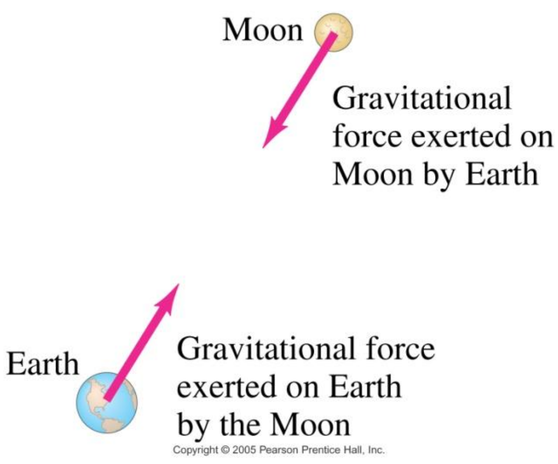
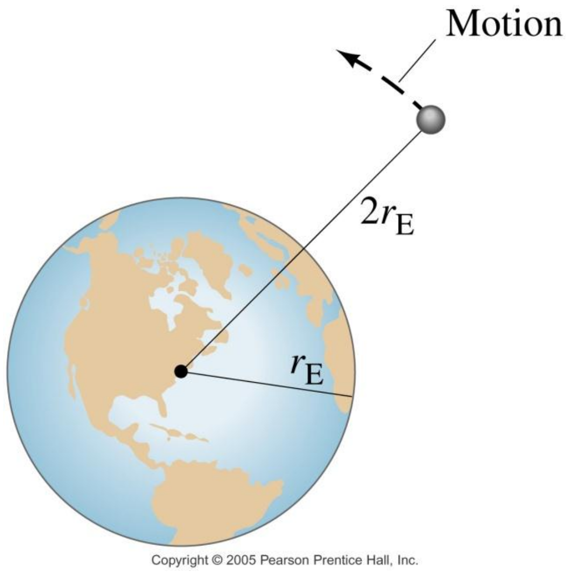
Chapter 5

Gravitation & Satellites

3 relationships in Ch 5 based on Newton’s Law of Gravitation

- gravitational attraction force
- orbital speed of satellites
- gravitational acceleration rate





Newton's Law of Universal Gravitation

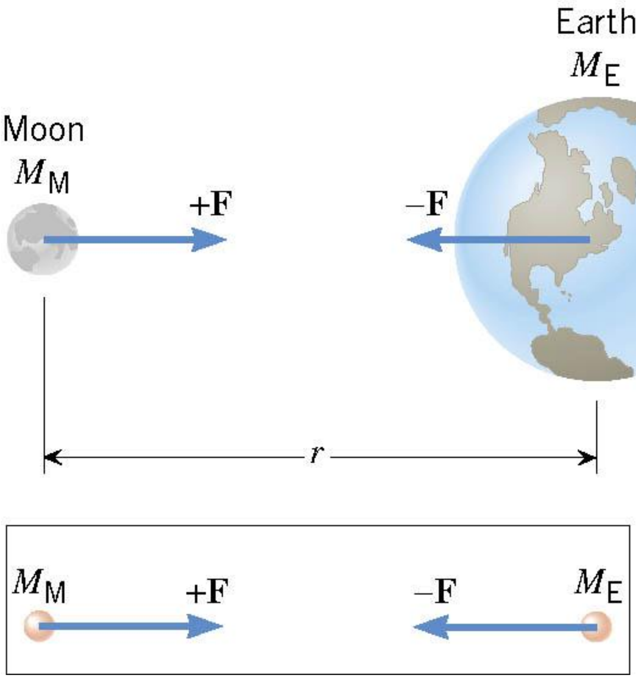
- gravitate: to move towards or be attracted to another object
- Two chunks of mass, m_1 m_2, that are separated by a distance r exert a gravitational attraction force F_g of equal magnitude on each other

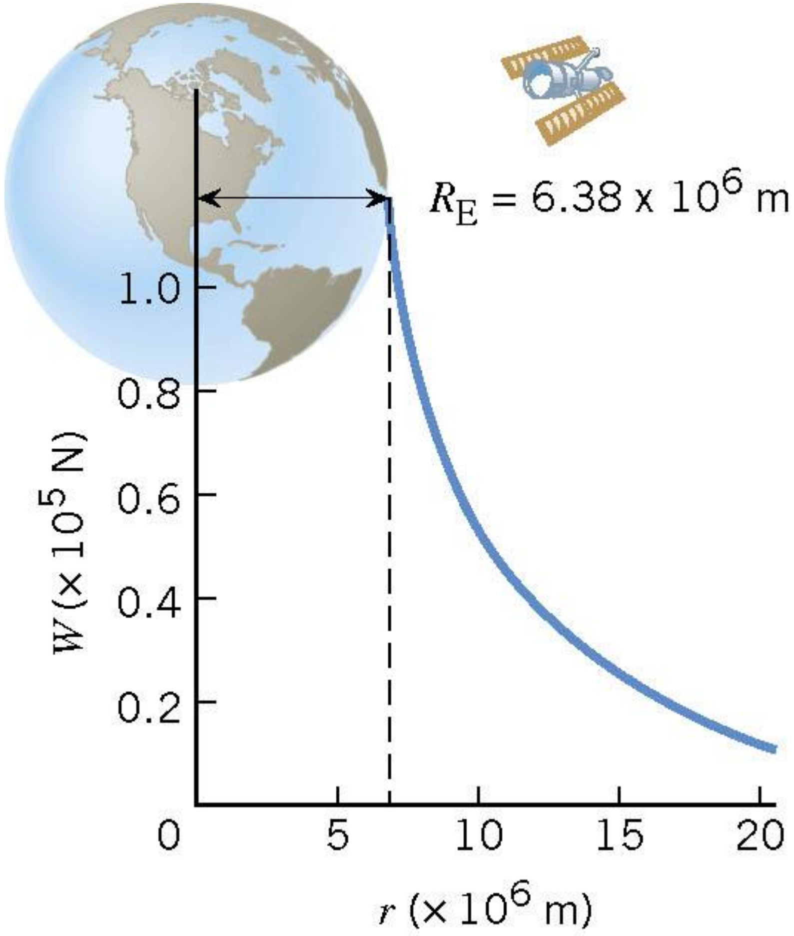
$$F_g = \frac{G \bullet m_1 \bullet m_2}{r^2} \quad G = 6.67 \times 10^{-11}\ \mathrm{Nm^2/kg^2}$$

- G is a universal proportionality constant that converts m_1m_2/r^2 into Newtons

- Magnitude of F_g is the same on both masses

Inverse Square Law





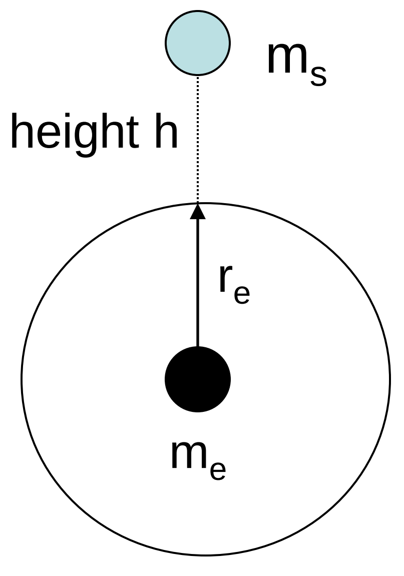
- force of equal magnitude acting on Moon and on Earth
- r is a center-to-center separation distance

F_g Calculations

Mass above Earth's surface (satellite or orbiting object)

Use r = (radius of Earth + height above surface)

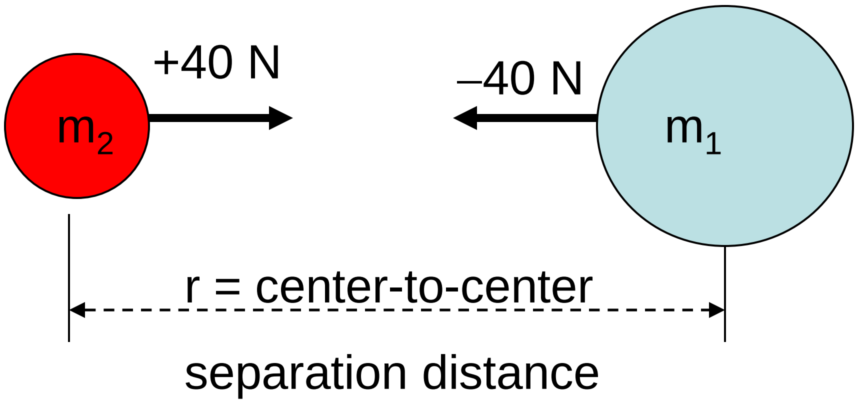
$F_g = Gm_sm_e/r^2$

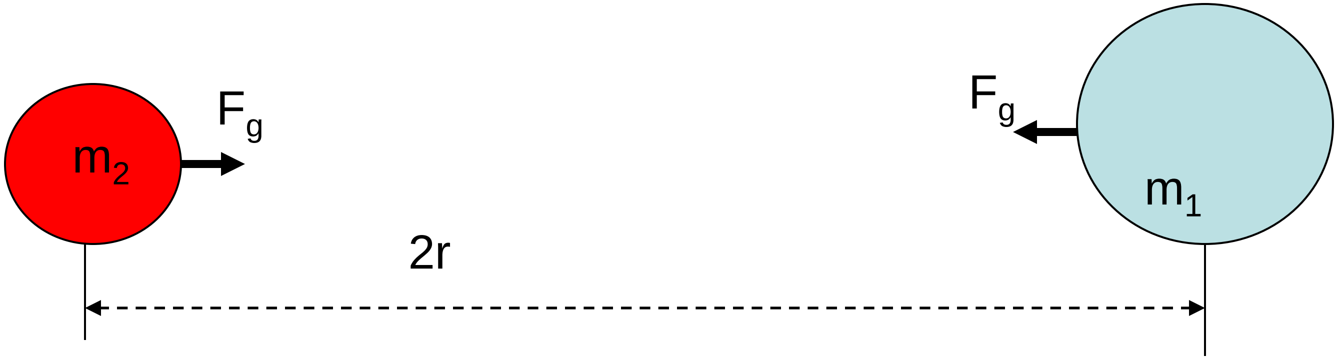


Earth's mass m_e concentrated at point in center

Newton and calculus

Practice with Universal Gravitation Law

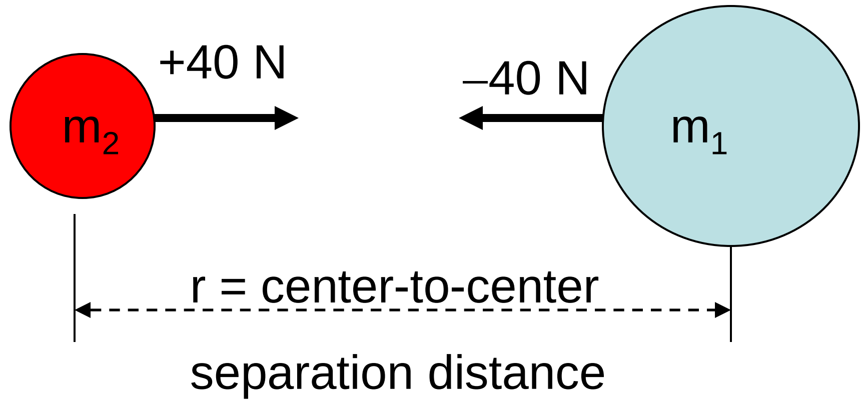


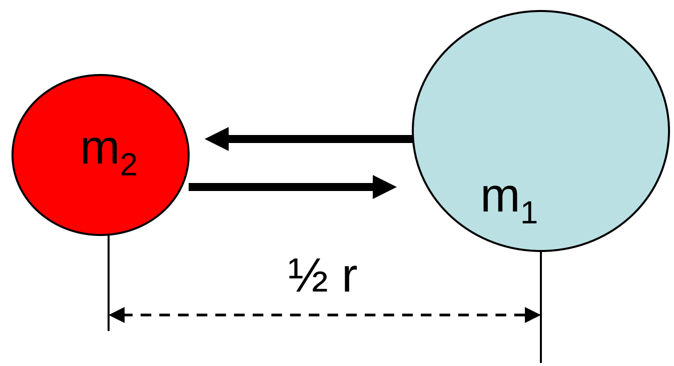


Double the separation distance to 2 r   New F_g value?

Fg = ¼ of original value (inverse square) = 10 N

Practice with Universal Gravitation Law

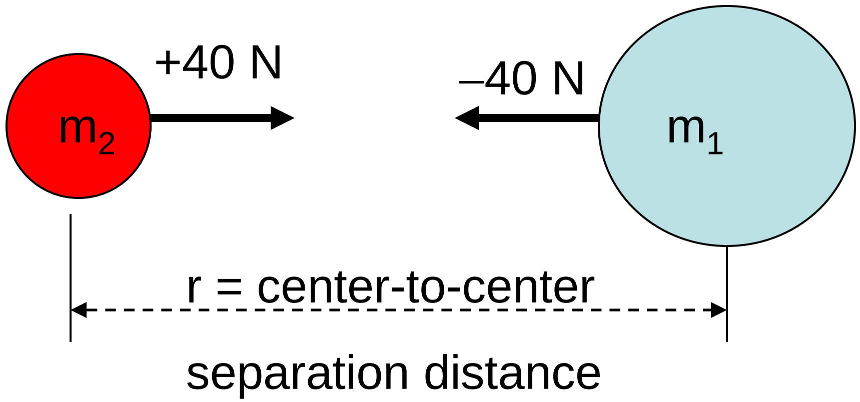


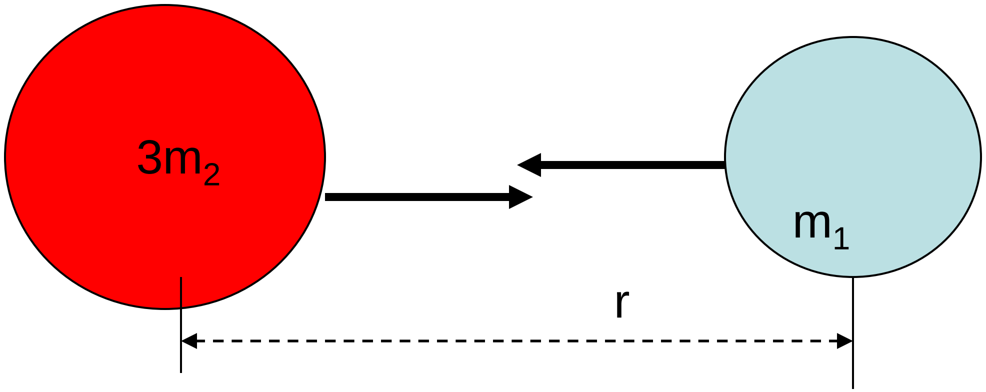


Reduce the separation distance to ½ r   New F_g value?

Fg = 4X original value (inverse square) = 160 N

Practice with Universal Gravitation Law



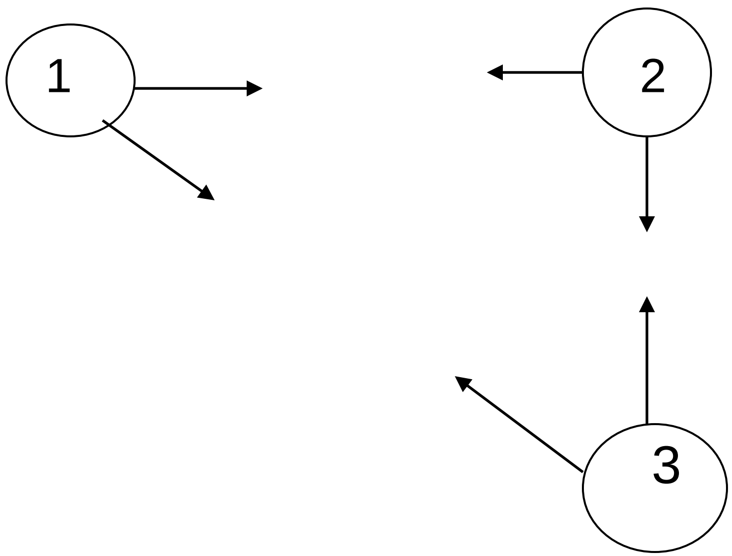


Triple the mass of m_2 to 3 m_2 New F_g value?

Fg = 3X original value (direct relationship) = 120 N

Superposition Principle

- F_g forces from adjacent masses do not interact or change each other

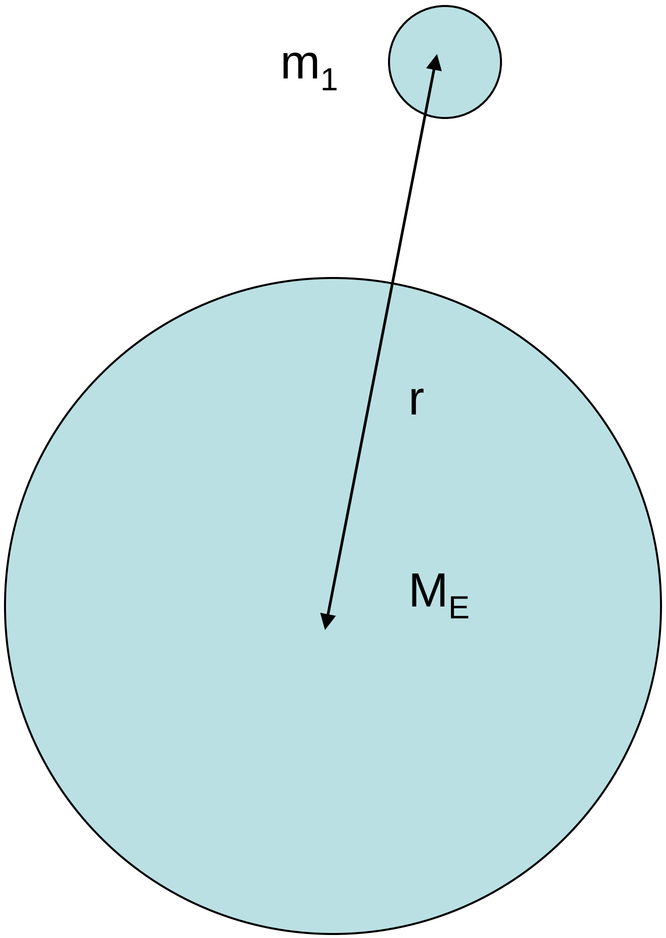


Calculate F_{12}, F_{13} separately and do a vector sum to find net force on 1

F_{23} does not change its magnitude

Gravitation near Earth's surface

- acceleration rate due to gravity as a function of r

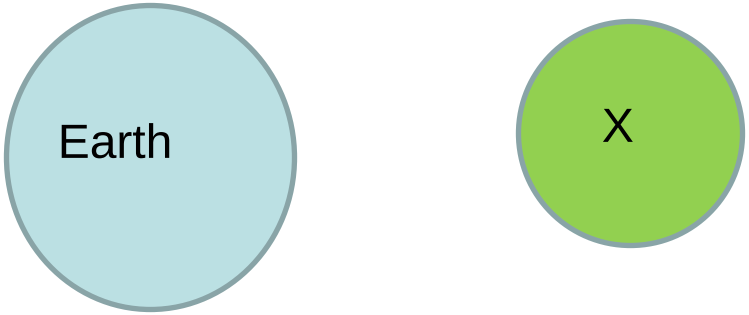


$$\Sigma F = m_1 a_g = G\frac{m_1 M_E}{r^2}$$

$$a_g = \frac{GM_E}{r^2}$$

acceleration rate is independent of the mass of the object

Variable change – prediction problems (usually M.C.)



$$a_g = \frac{GM_E}{r^2}$$

M_E, R_E, g_E

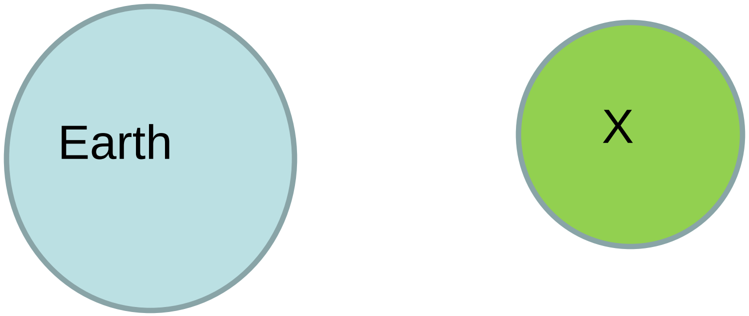
$M_X = 2M_E$ $R_x = R_E$ $g_X = ?$ — $2g_E$

$M_X = M_E$ $R_X = ½ R_E$ $g_X = ?$ — $4\ g_E$

$M_X = ½\ M_E$ & $R_X = 2R_E$ $g_X = ?$ — $1/8\ g_E$

Variable change – prediction problems

$$a_g = \frac{GM_E}{r^2}$$



$g_X = g_E$

$M_X = 2M_E$ $R_X = ?$ $$R_X = \sqrt{2}\, R_E$$

$R_X = 2\,R_E$ $M_X = ?$ $4\,M_E$

Deriving UCM velocity

- Determine what force(s) are providing the centripetal force
- Equate that force with F_c
- solve for v

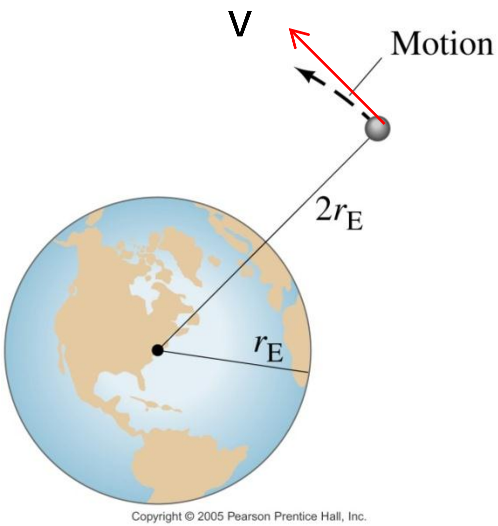
$$F_c = F_g$$

$$\frac{m_S v^2}{r} = \frac{G m_E m_S}{r^2}$$

$$v = \sqrt{\frac{G m_E}{r}}$$

satellite speed independent of its mass

Variable change – prediction problems



for v' = 2v   r'=? $$r' = \frac{r}{4}$$

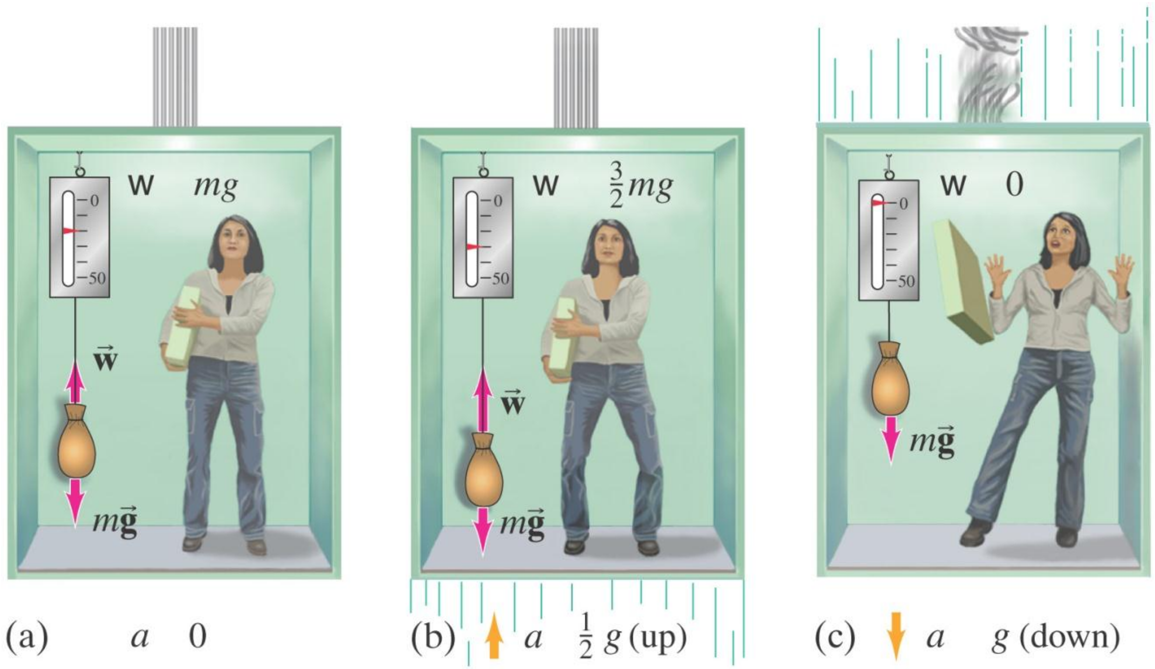
$$v = \sqrt{\frac{Gm_E}{r}}$$

$m_s' = 2m_s$ v'=?   v' = v

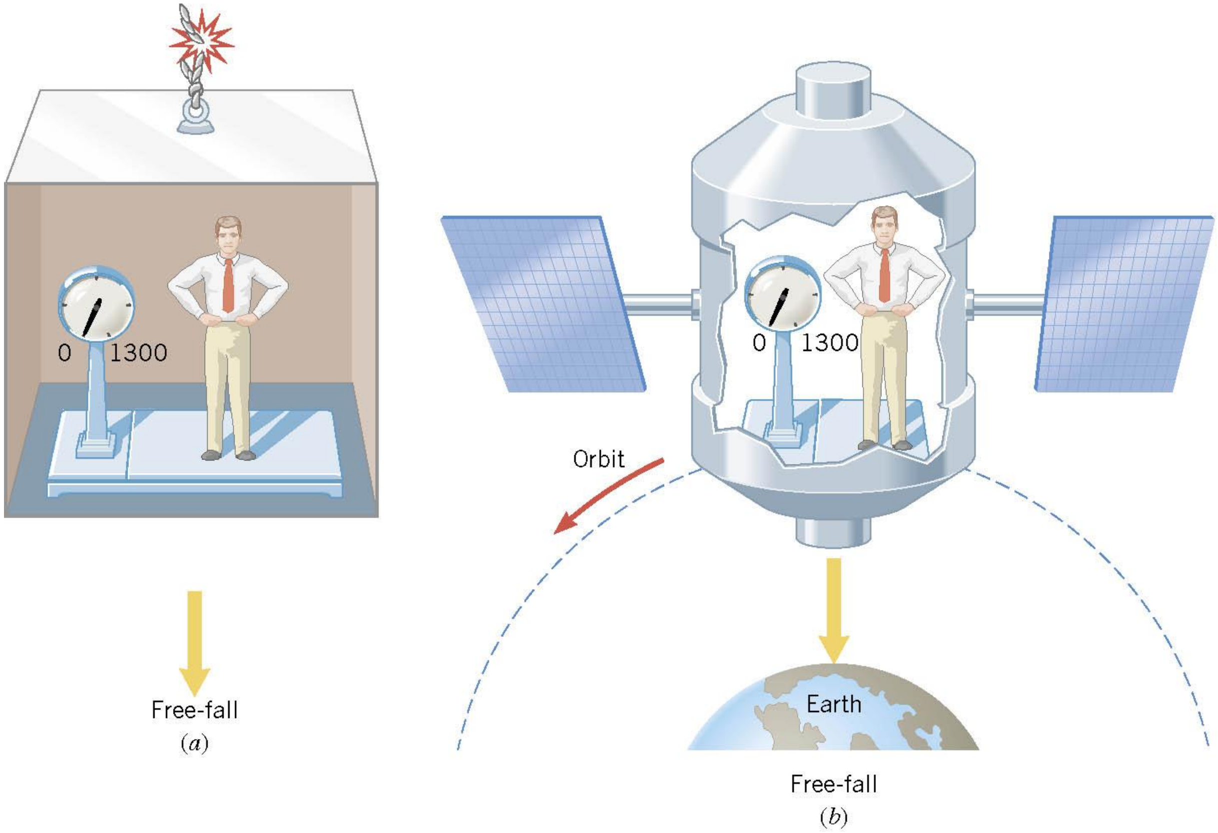
5-8 Satellites and "Weightlessness"

Objects in orbit are said to experience **weightlessness**. They do have a gravitational force acting on them, though!

The satellite and all its contents are in **free fall**, so there is no **normal force**. This is what leads to the experience of weightlessness.



Apparent weightlessness in orbit



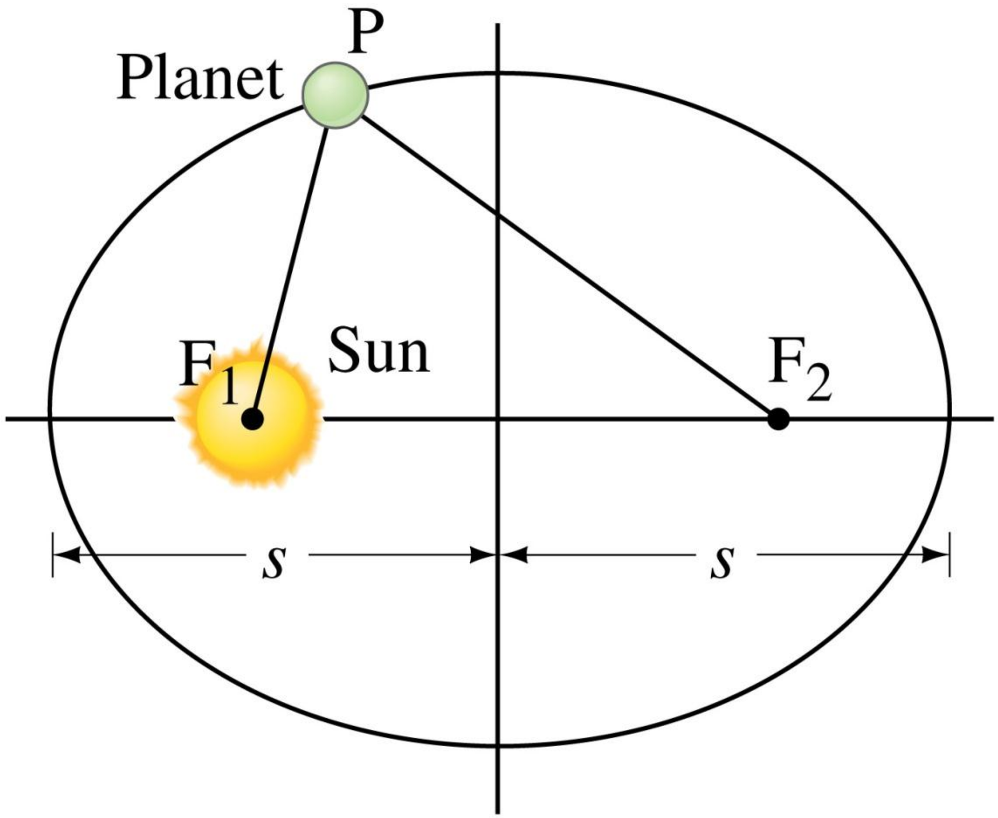
Apparent Weightlessness

- Spacecraft and all contents are in a state of continuous free-fall towards Earth
- Since cabin floor is “falling” at same rate as astronaut, the floor cannot exert a normal force up on astronaut
- Centripetal acceleration = free-fall acceleration rate (approx 5 – 8 m/s^2 in orbits around Earth)
- Centripetal force provided by Fg – gravitational attraction force
- ***NOT*** because weight = 0
- Mr. Connell video

5-9 Kepler's Laws

Kepler's laws describe planetary motion.

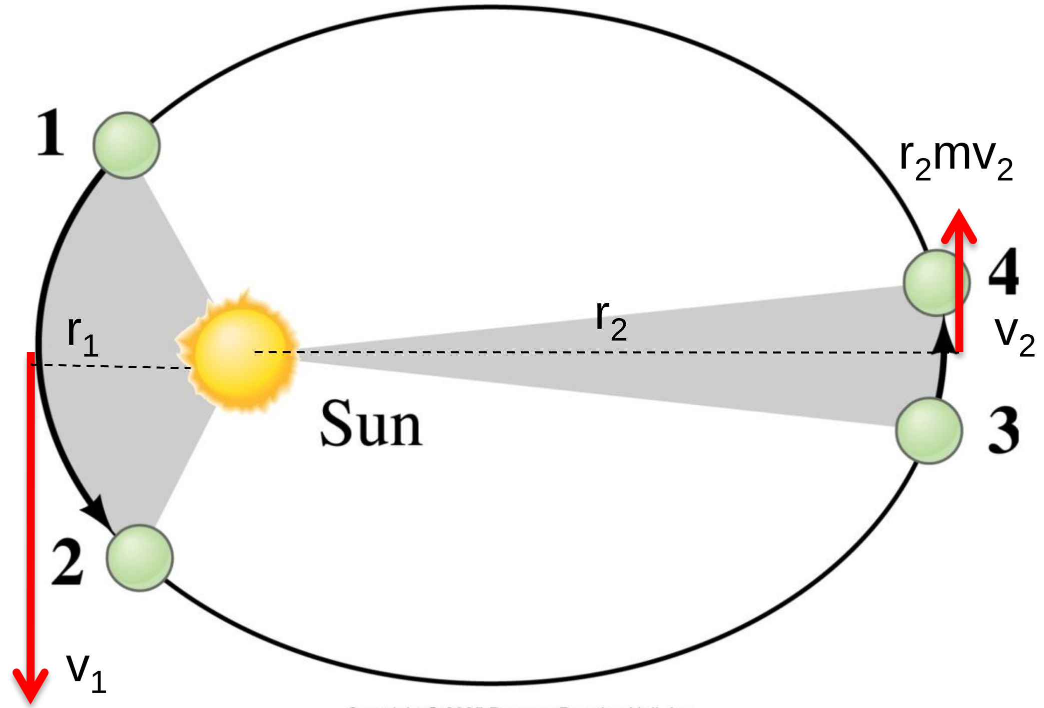
1. The orbit of each planet is an **ellipse**, with the Sun at one focus.



5-9 Kepler's Laws and Newton's Synthesis

2. An imaginary line drawn from each planet to the Sun sweeps out equal areas in equal times.

smaller radius orbit causes faster orbital speed



larger radius orbit causes slower orbital speed

Law of Areas equivalent to angular momentum conservation

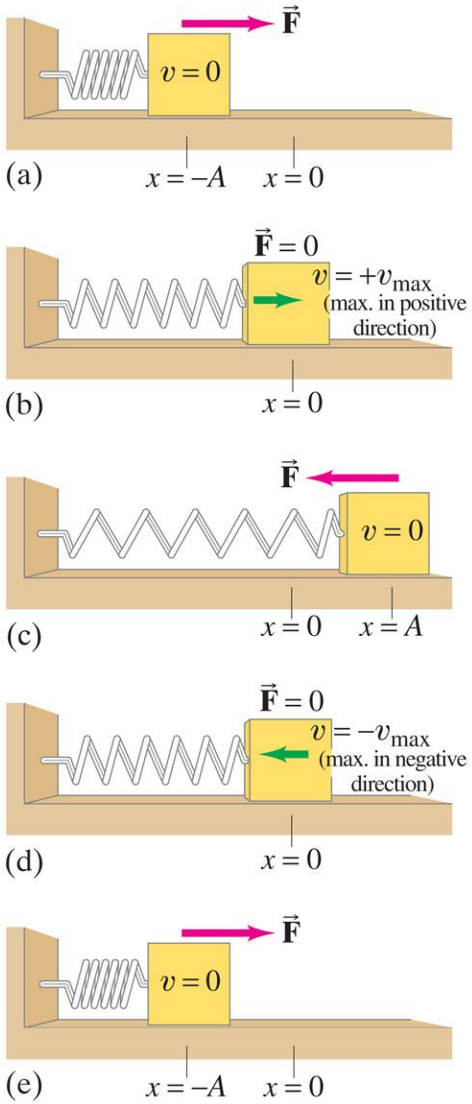
$r_1mv_1 = r_2mv_2$

Chapter 11
Simple Harmonic Motion

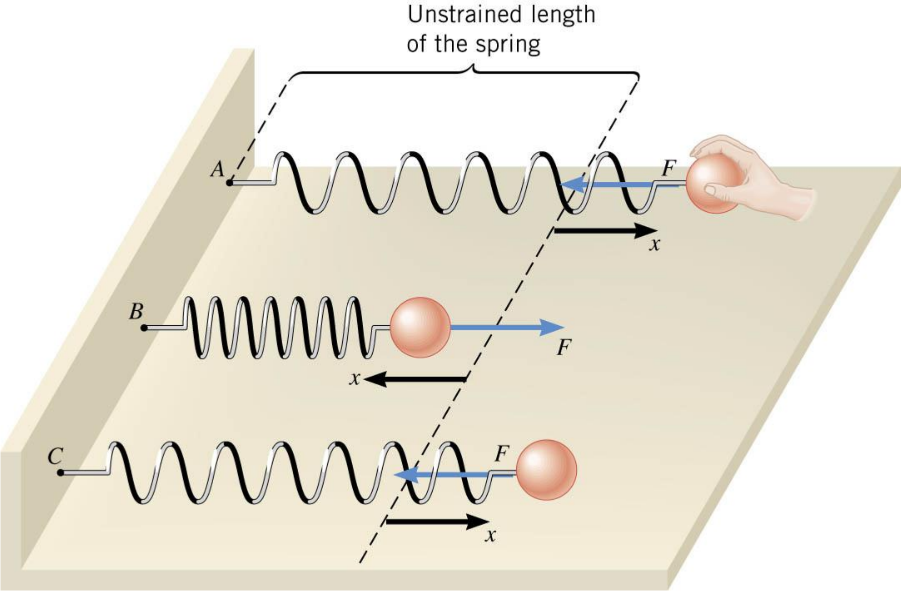
Simple Harmonic Motion

- A: amplitude = max displacement
- x = 0: equilibrium where ΣF = 0
- k: spring constant Hooke's law

$$k = \frac{F}{x}$$



Restoring Force points to center

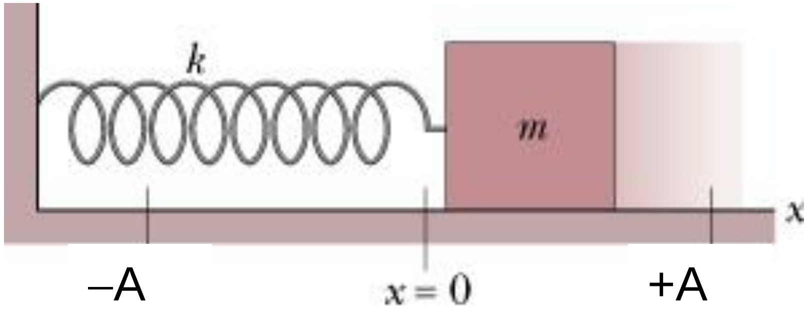


Restoring force $F = -kx$

- directed opposite the displacement
- directly proportional to displacement
- maximum at the amplitudes

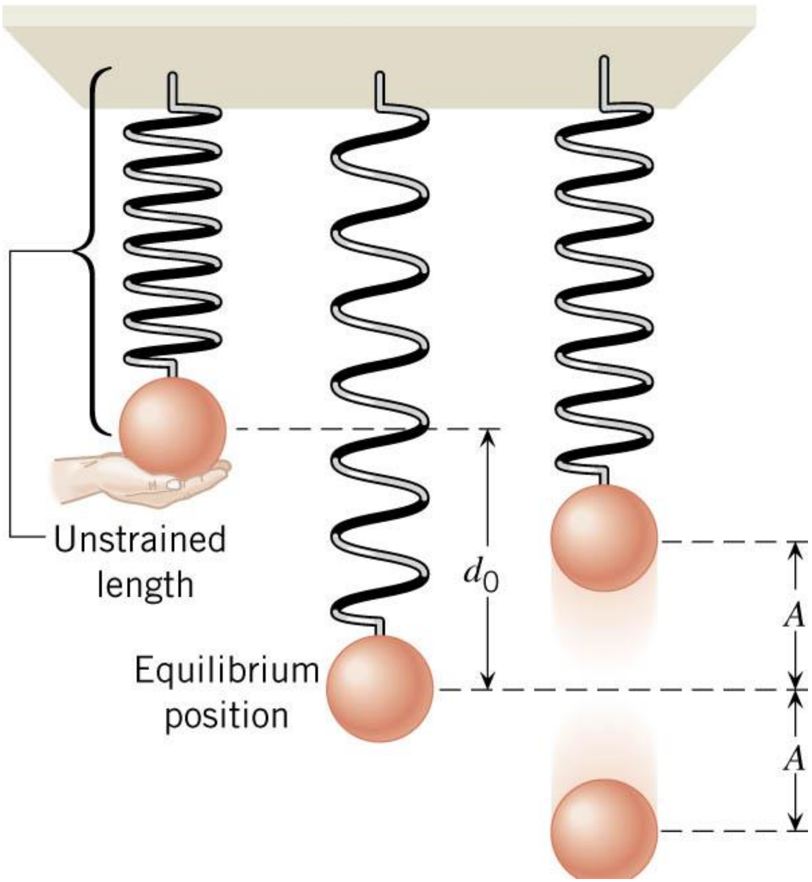
  $F = \pm kA$

SHM quantities max/zero



Displacement	max	0	max
Speed	0	max	0
Acceleration	max	0	max
Restoring Force	max	0	max

Vertical Springs



- mass is hung from spring at rest at unstrained length
- calculate k using equilibrium
- spring force = kd_o = mg
- oscillation occurs above and below this equilibrium position
- amplitude is then the max displacement from this equilibrium position, determined by person/conditions which create oscillation

Frequency & Period

- frequency f: $$\frac{\#\ \text{of}\ oscillations}{1\ \sec ond}$$
  - SI units are Hertz (Hz)

inversely related

$$T = \frac{1}{f} \qquad f = \frac{1}{T}$$

- period T: $$\frac{\#\ \text{of seconds}}{1\ \text{full oscillation}}$$

SHM is sinusoidal motion

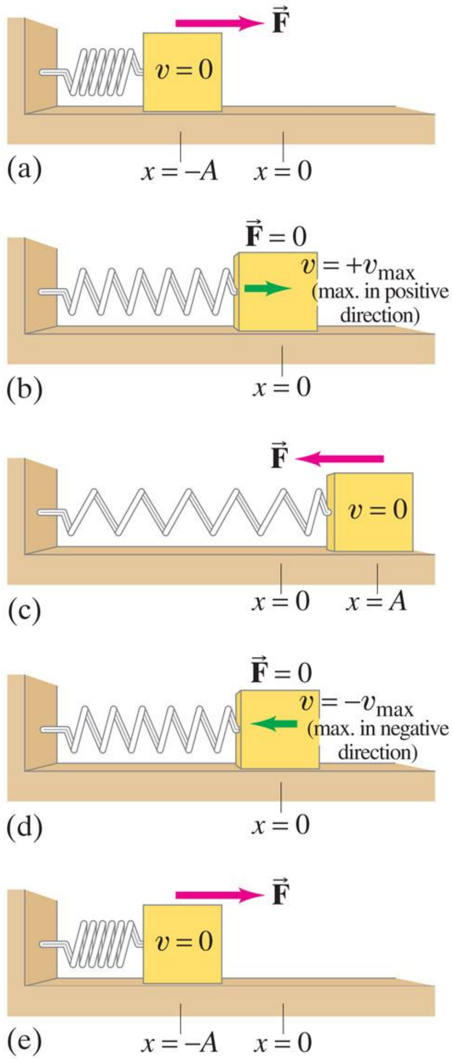
t = 0

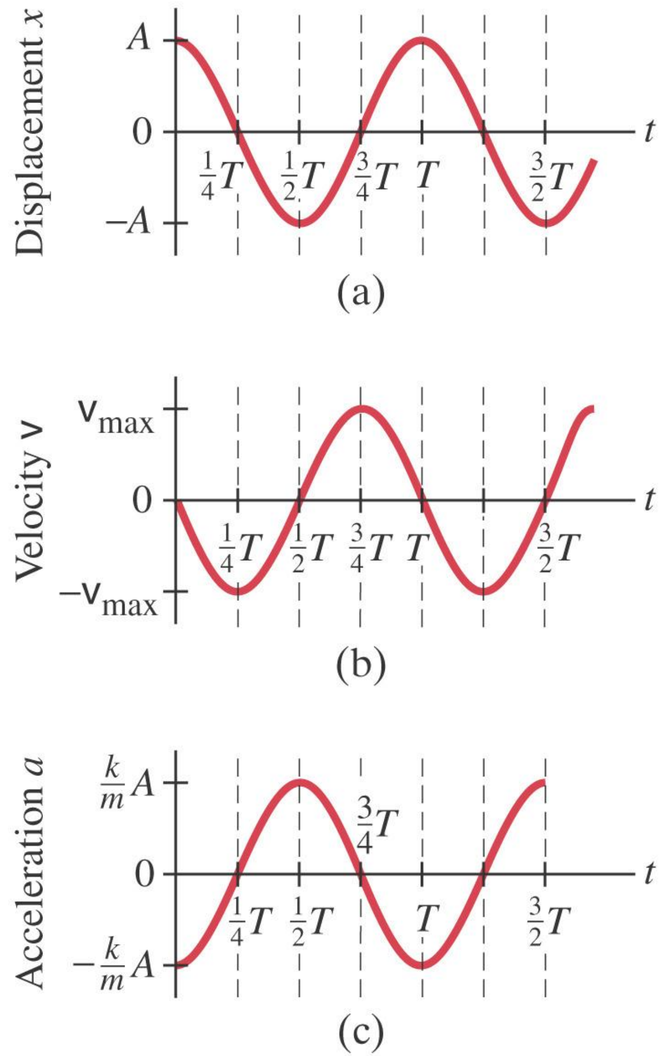
t = T/4

t = T/2

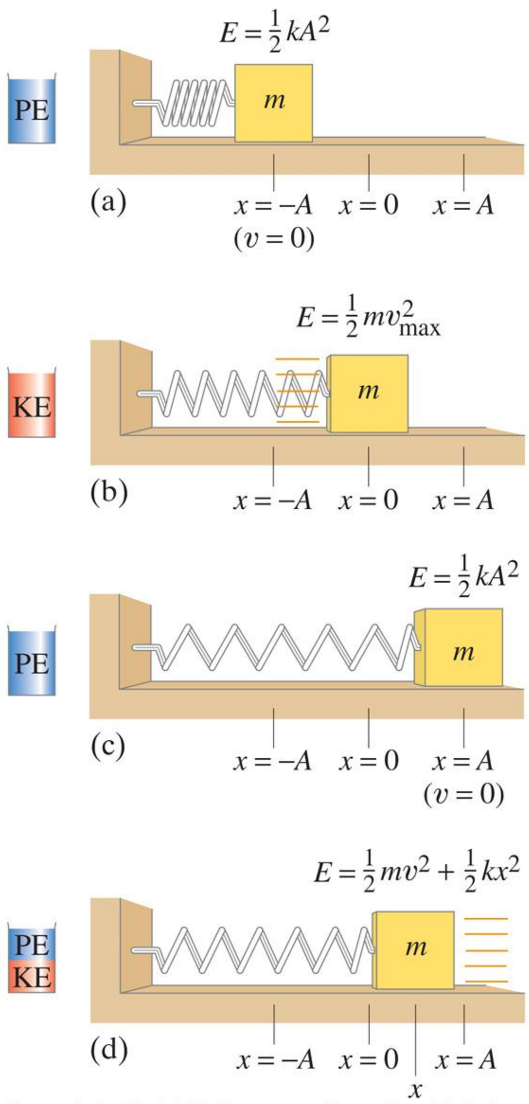
t = ¾ T

t = T





Energy approach to SHM



Total mechanical energy stays constant

$E = KE + PE = ½\ mv^2 + ½\ kx^2$

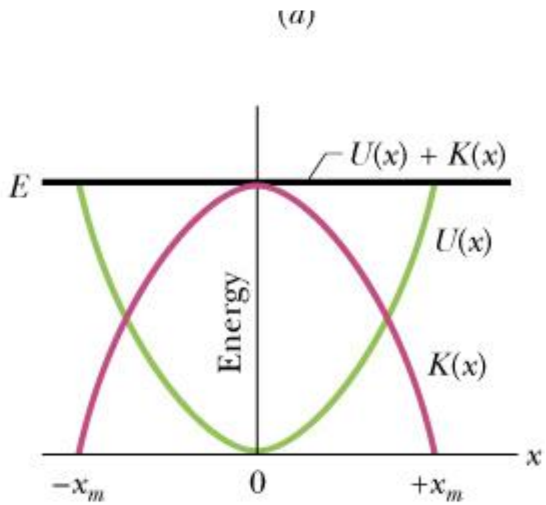
Total energy = all PE = $½\ kA^2$ at either amplitude position

OR

Total energy = all KE = $½\ mv_{max}^2$ at equilibrium position

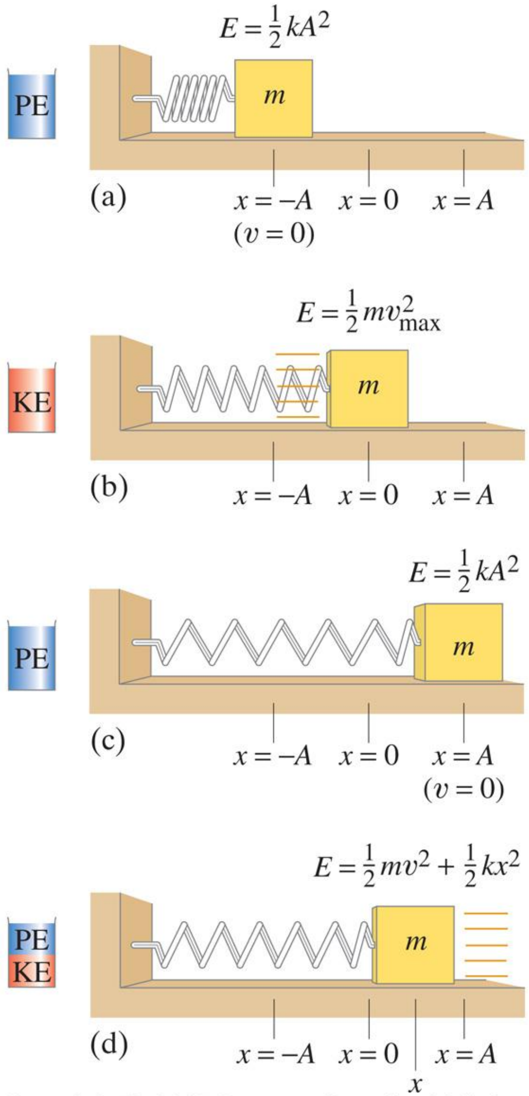
Energy in SHM – graphical approach

Elastic Potential Energy U
Kinetic Energy K
Total mechanical energy E



(b)

x, v, a calculations



1) displacement or velocity

$\frac{1}{2} kA^2 = \frac{1}{2} mv^2 + \frac{1}{2} kx^2$

2) max speed (moving + or –)

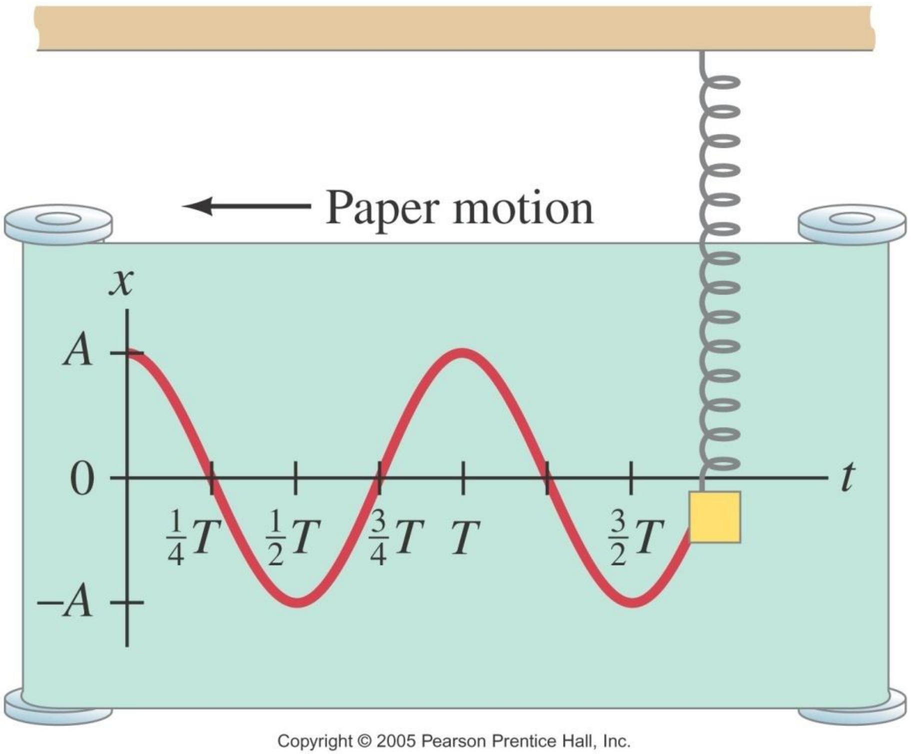
$\frac{1}{2} kA^2 = \frac{1}{2} mv_{max}^2$

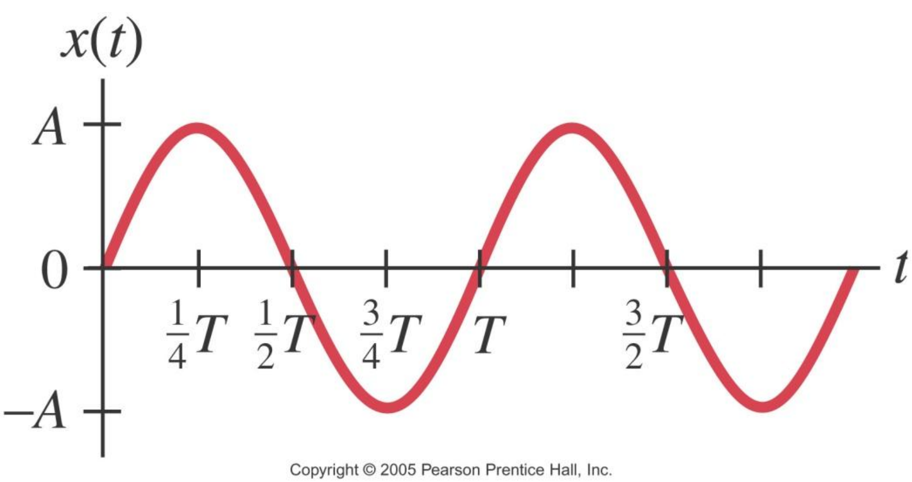
3) max acceleration

$a_{max} = F_{max}/m = kA/m$

CAUTION: derived formulas not on formula sheet!!

11-3 The Period and Sinusoidal Nature of SHM





SHM is motion that varies sinusoidally with time

The bottom curve is the same, but shifted ¼ period so that it is a sine function rather than a cosine.

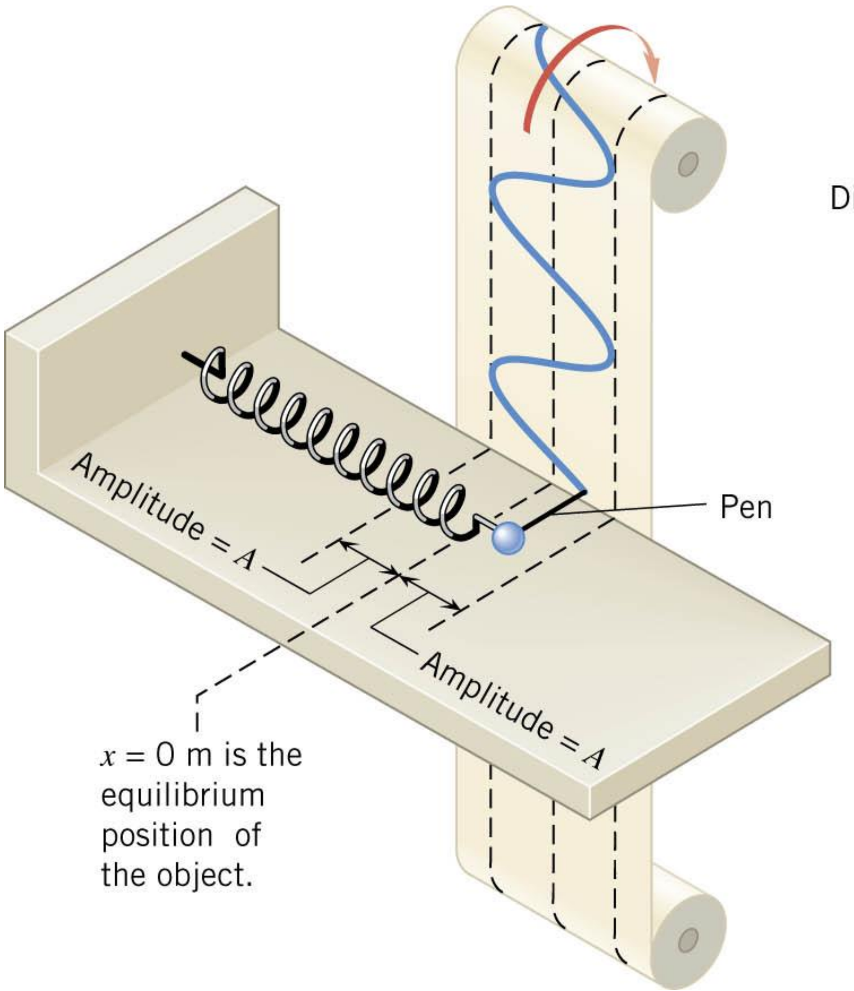
2 functions to recognize and understand

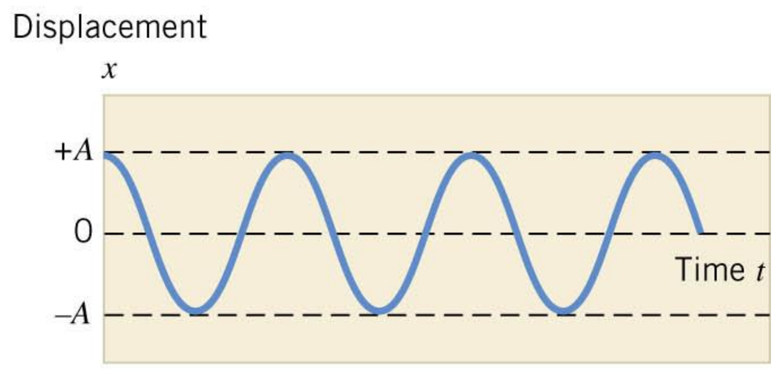
$x(t) = A\cos(\omega t)$

or

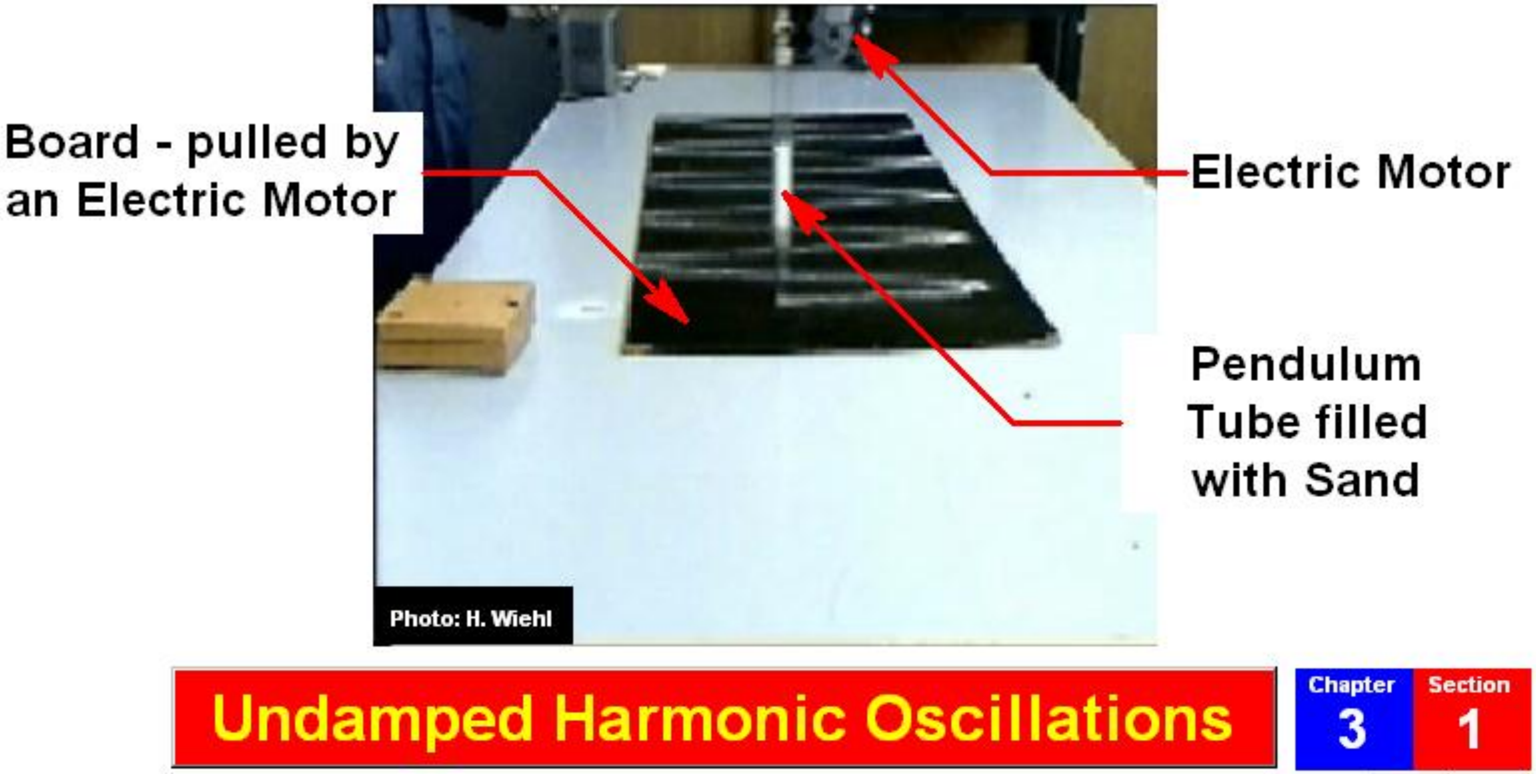
$x(t) = A\sin(\omega t)$

SHM – sinusoidal functions





sinusoidal variation with time NOT linear, quadratic, inverse or exponential



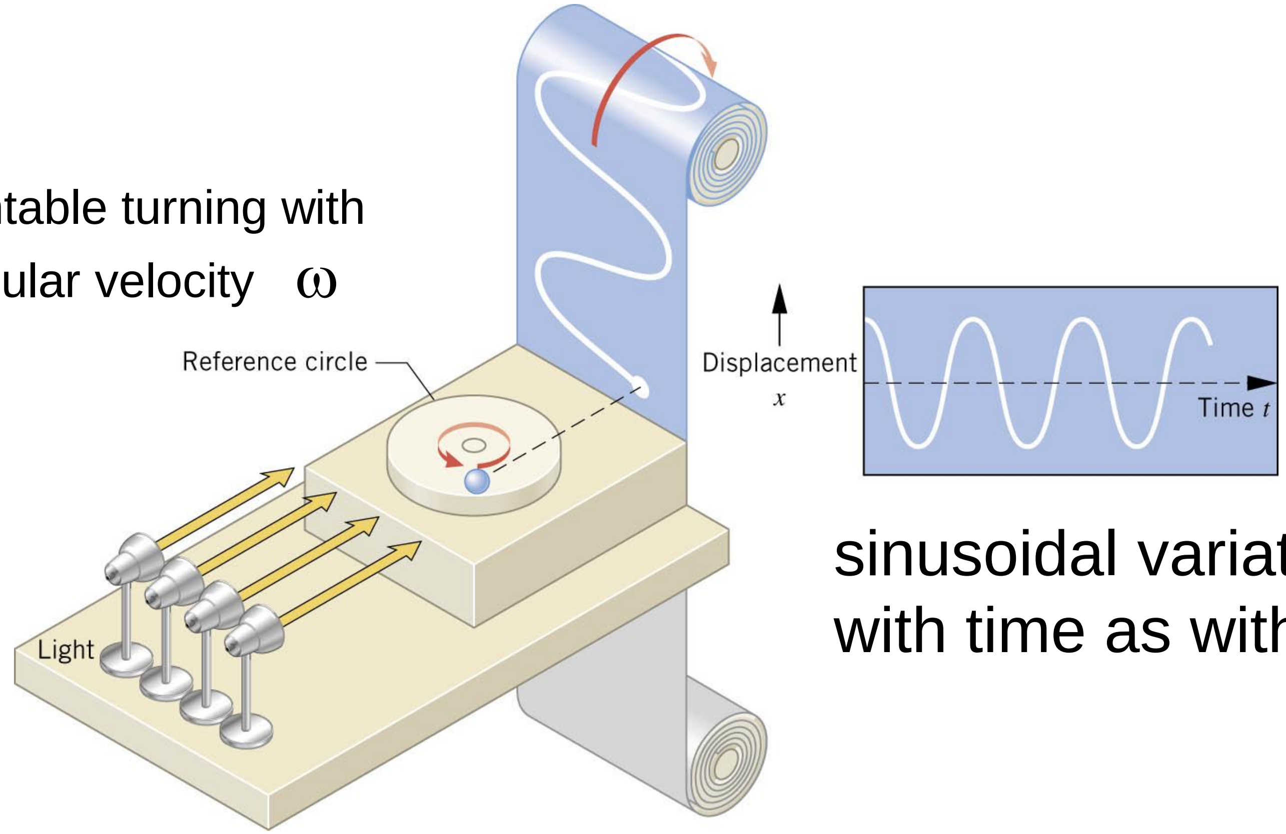
Board - pulled by an Electric Motor
Electric Motor
Pendulum Tube filled with Sand
Photo: H. Wiehl
Undamped Harmonic Oscillations
Chapter 3
Section 1

demo

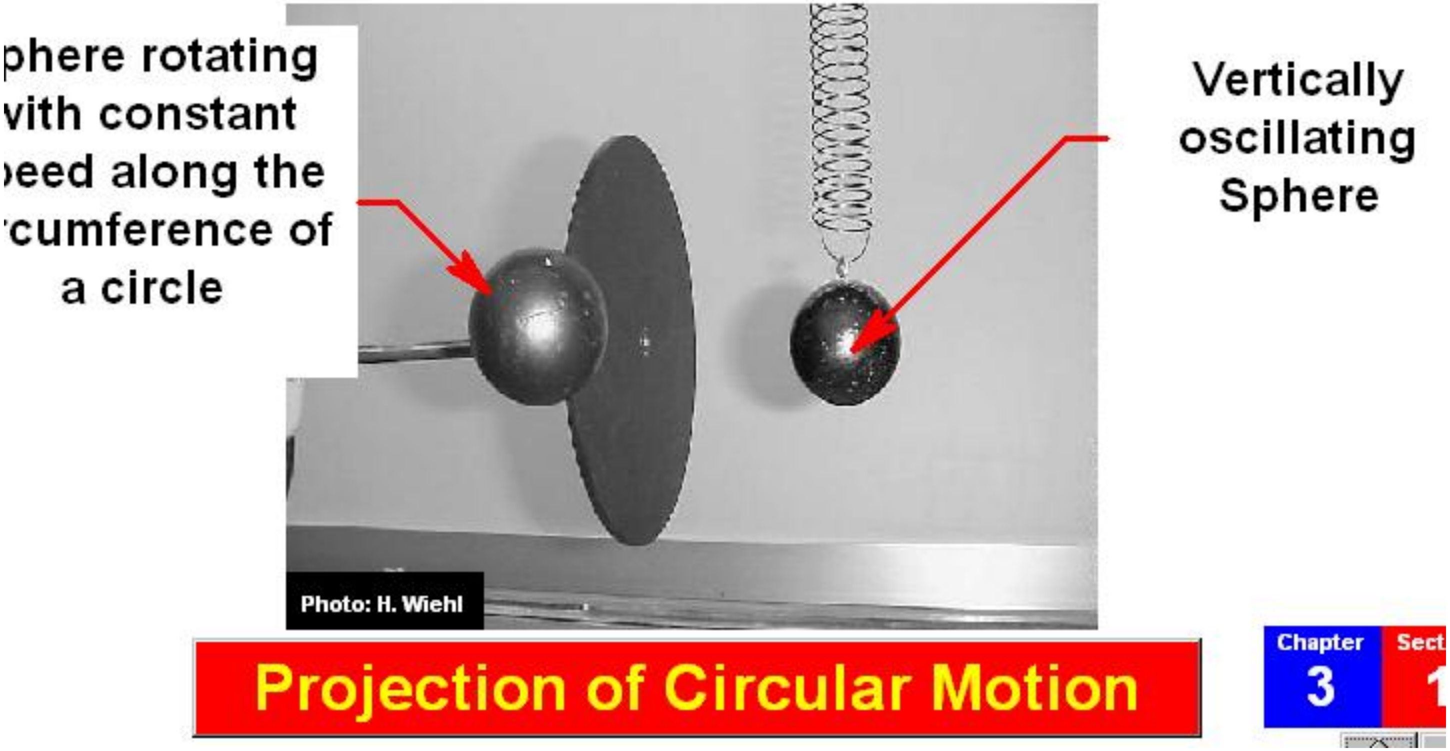
UCM – SHM relationship

shadow projection of UCM creates SHM

turntable turning with
angular velocity ω



sinusoidal variation
with time as with spring


phere rotating
vith constant
eed along the
cumference of
a circle
Vertically
oscillating
Sphere
Photo: H. Wiehl
Projection of Circular Motion
Chapter
3
Sect

demo

ω   omega

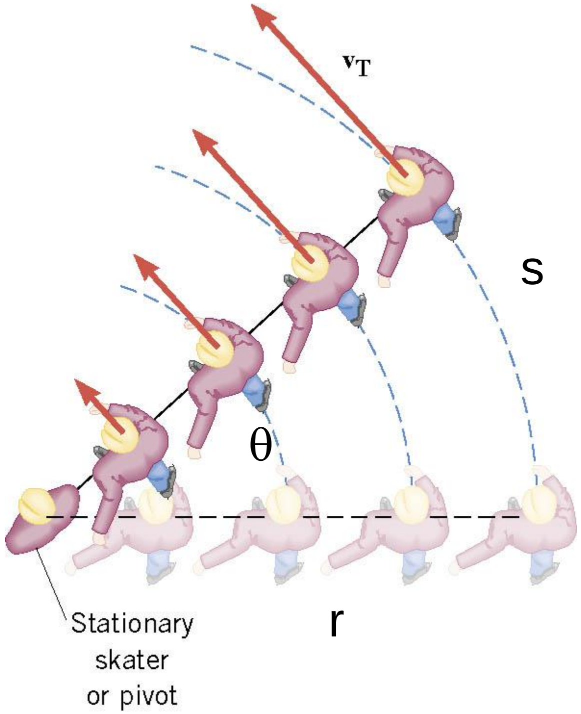
- Linear velocity

$$v_T = \frac{s}{t}$$

- Angular velocity

$$\omega = \frac{\theta \text{ in } radians}{t \text{ in } seconds}$$

- θ is the same for all skaters; ω is the same for all skaters



- ω of the UCM object can be used to locate the SHM object

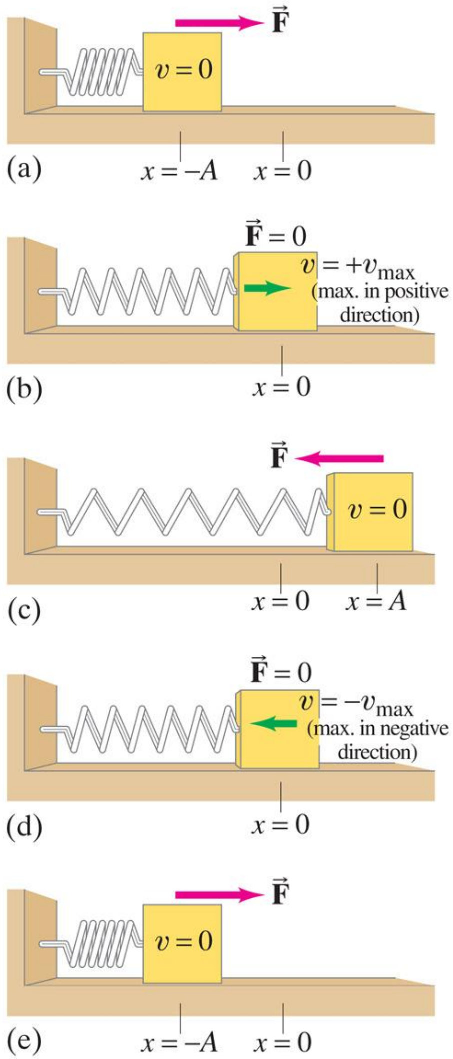
Angular Velocity – Angular Frequency ω

- since $$\omega = \frac{\theta}{t} \text{ then } \theta = \omega t$$
- angular frequency ω (omega)
  - same as the angular velocity of an object in UCM in radians per second

$$\omega = \frac{2\pi \text{ radians}}{T \text{ seconds}} = 2\pi f$$

f must be in Hertz

each cycle of an object in SHM consists of angular displacement = 2 π radians



0 radians

π/2 radians

π radians

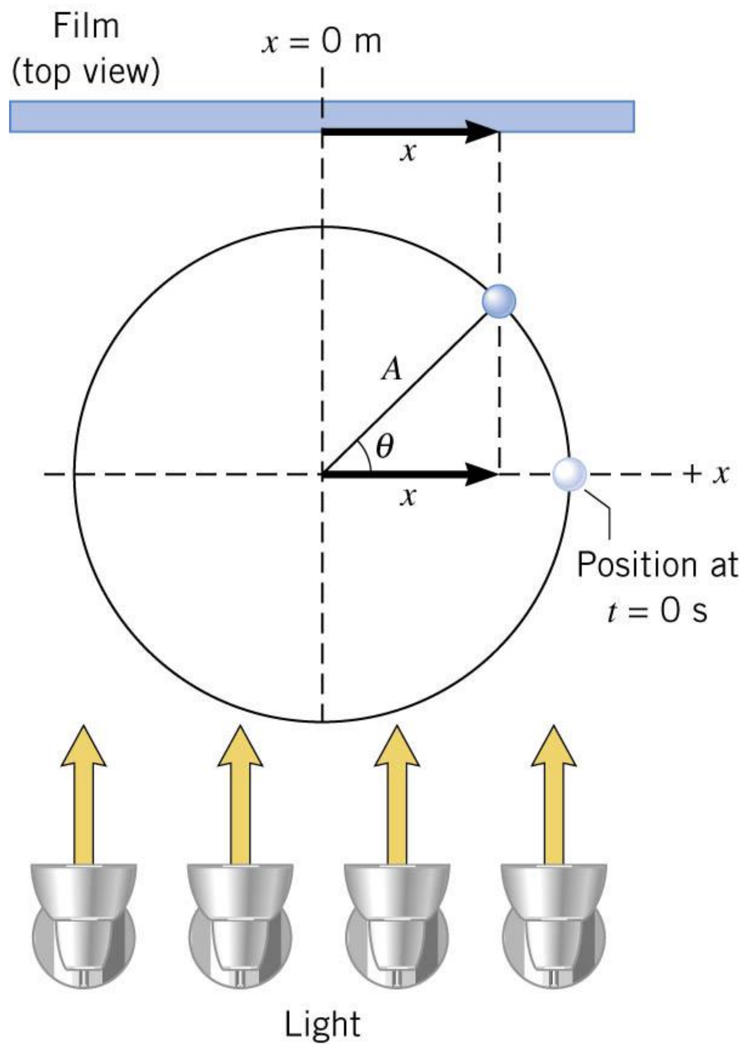
3 π/2 radians

2 π radians

Displacement – time function

$$x = A\cos(\theta)$$

$$\theta = \omega t$$



Maximum Velocity

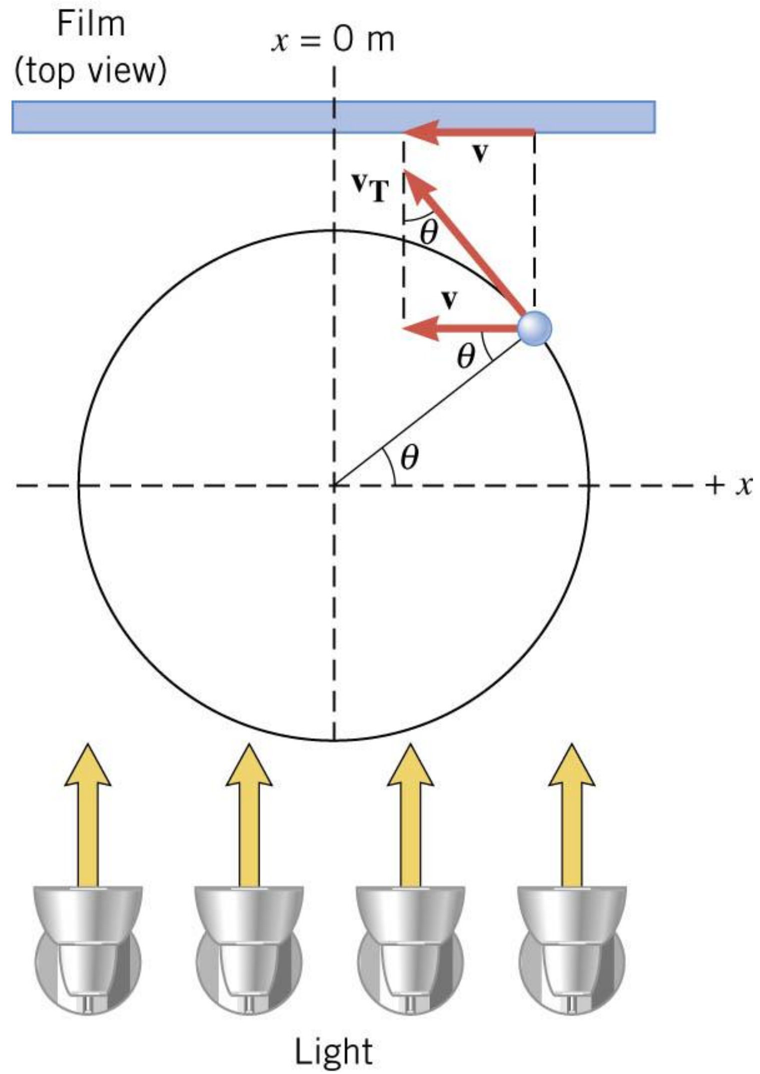
radius of UCM circle =A

UCM speed = SHM max speed

$$v_T = \frac{2\pi r}{T} = \frac{2\pi A}{T}$$

therefore

$$v_{\max} = \omega A$$



Period of SHM

- Period of an object (mass on a spring or a pendulum) is independent of the amplitude of the motion
  - with greater amplitude comes greater restoring force
  - greater restoring force causes faster speeds
  - object covers the longer distance at faster speeds
  - period stays constant

Period for mass on a spring

$$v_{\max} = \frac{2\pi A}{T} \; from\, UCM$$

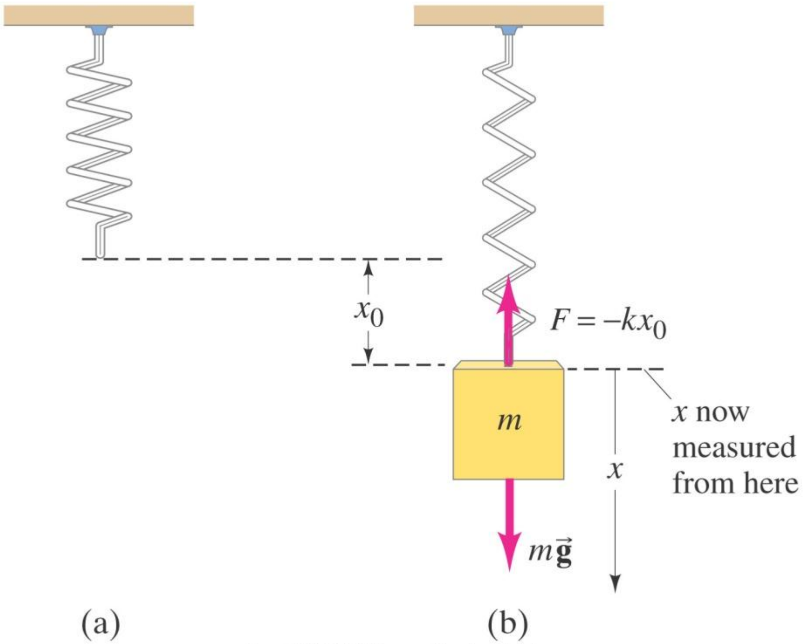
from energy approach

$$\frac{1}{2}kA^2 = \frac{1}{2}mv_{\max}{}^2$$

$$v_{\max} = \sqrt{\frac{k}{m}}A$$

$$\frac{2\pi A}{T} = \sqrt{\frac{k}{m}}A$$

$$T = 2\pi\sqrt{\frac{m}{k}}$$



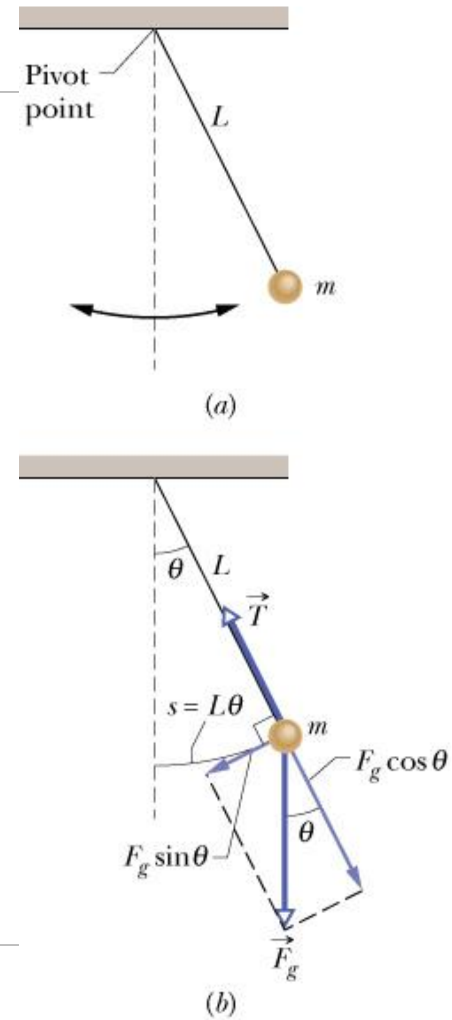
Mass on a spring

- Same formula for horizontal or vertical spring

$$T = 2\pi\sqrt{\frac{m}{k}}$$

- Period depends on:
  - mass of object
  - spring constant
- Period is independent of:
  - gravity
  - amplitude

Simple pendulum



- Bob at the end of a string

$$T = 2\pi\sqrt{\frac{L}{g}}$$

- Period depends on:
  - length of pendulum
  - gravitational acceleration rate
- Period is independent of:
  - mass of the bob
  - amplitude